Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use letter symbols and distinguish their different roles in algebra

Use, read and write, spelling correctly:
algebra, unknown, symbol, variable… equals… brackets…
evaluate, simplify, substitute, solve…
term, expression, equation… squared… commutative…

Reinforce the idea of an **unknown**. Answer questions such as:

- $5 + \square = 17$
- $3 \times \square - 5 = 7$
- $\blacktriangledown + \blacklozenge = 4$. What numbers could $\blacktriangledown$ and $\blacklozenge$ be?
- The product of two numbers is 24. What could they be?

Know that letters are used to stand for numbers in algebra.
Begin to distinguish between different uses of letters.
For example:

- In the equation $3n + 2 = 11$, n is a particular unknown number, but in the equation $a + b = 12$, a and b can take many different values.

Recognise algebraic conventions, such as:

- $3 \times n$ or $n \times 3$ can be thought of as '3 lots of n', or $n + n + n$, and can be shortened to $3n$.
- In the expression $3n$, n can take any value, but when the value of an expression is known, an equation is formed, i.e. if $3n$ is 18, the equation is written as $3n = 18$.

Understand the meaning of and begin to **use simple expressions with brackets,** e.g. $3(n + 2)$ meaning $3 \times (n + 2)$, where the addition operation is to be performed first and the result of this is then multiplied by 3.

Use the equals sign appropriately and correctly.

- Recognise that if $a = b$ then $b = a$, and that $a + b = c$ can also be written as $c = a + b$.
- Avoid errors arising from misuse of the sign when setting out the steps in a calculation, e.g. incorrectly writing
  $38 + 29 = 38 + 30 = 68 - 1 = 67$

Use letter symbols to write expressions in meaningful contexts.
For example:

add 7 to a number	$n + 7$
subtract 4 from a number	$n - 4$
4 minus a number	$4 - n$
a number multiplied by 2 and then 5 added	$(n \times 2) + 5$ or $2n + 5$
a number divided by 2	$n \div 2$ or $n/2$
a number plus 7 and then multiplied by 10	$(n + 7) \times 10$ or $10(n + 7)$
a number multiplied by itself	$n \times n$ or n^2

Understand the difference between expressions such as:

$2n$ and $n + 2$	$3(c + 5)$ and $3c + 5$
n^2 and $2n$	$2n^2$ and $(2n)^2$

Link to formulating expressions and formulae (pages 122–5).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
algebraic expression, formula, function...
partition, multiply out... cubed, to the power of...

Know that an *algebraic expression* is formed from letter symbols and numbers, combined with operation signs such as +, –, ×, ( ), ÷ and /.

Use letter symbols in different ways and begin to distinguish their different roles. For example:

- In the equation $4x + 3 = 47$, x is a particular unknown number.
- In the formula $V = IR$, V, I and R are variable quantities, related by the formula.
  In the formula $F = {}^{9C}\!/_{5} + 32$, once C is known, the value of F can be calculated.
- In the function $y = 3x – 4$, for any chosen value of x, the related value of y can be calculated.

Know how multiplication and division are represented in algebraic expressions. For example:

- $2 \times n$ is written as $2n$.
- $p \times q$ is written as pq.
- $a \times (b + c)$ is written as $a(b + c)$.
- $(x + y) \div z$ is written as $\dfrac{x + y}{z}$.

Use the equals sign appropriately and correctly.

- Know that the symbol = denotes equality.
- Avoid misuse of the equals sign when working through a sequence of steps, e.g. incorrectly writing $56 + 37 = 56 + 30 = 86 + 7 = 93$.
- Avoid interpreting the equals sign as 'makes', which suggests it means merely the answer to a calculation, as in $3 \times 2 + 7 = 13$.

Begin to interpret the equals sign more broadly, including in equations with expressions on each side. For example:

- Recognise equalities in different forms, such as:
  $a + b = c + d$ $\qquad 8 = 15 – 3x$
  and know that they can be written as:
  $a + b = c + d$ or $c + d = a + b$
  $8 = 15 – 3x$ or $15 – 3x = 8$
- Know that expressions such as $2a + 2$ and $2(a + 1)$ always have the same value for any value of a.

Link to constructing and solving equations (pages 122–5).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
identity, identically equal, inequality... subject of the formula... common factor, factorise... index law...
linear, quadratic, cubic... and the identity sign (≡).

Explain the distinction between equations, formulae and functions. For example:

- In the equation $5x + 4 = 2x + 31$, x is a particular unknown number.
- In the formula $v = u + at$, v, u, a and t are variable quantities, related by the formula. Once the values of three of the variables are known, the fourth value can be calculated.
- In the function $y = 8x + 11$, for any chosen value of x, the related value of y can be calculated.

Know that an inequality or ordering is a statement that one expression is greater or less than another. For example:

- $x \geq 1$ means that x is greater than or equal to 1.
- $y \leq 2$ means that y is less than or equal to 2.

An inequality remains true if the same number is added to or subtracted from each side, or if both sides are multiplied or divided by the same positive number. Multiplying or dividing by a negative number reverses the sense of the inequality.

Know the meaning of an identity and use the ≡ sign.
In an identity, the expressions on each side of the equation always take the same value, whatever numbers are substituted for the letters in them; the expressions are said to be identically equal.
For example:

- $4(a + 1) \equiv 4a + 4$ is an identity, because the expressions $4(a + 1)$ and $4a + 4$ always have the same value, whatever value a takes.

Link to constructing and solving equations (pages 122–9).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Know that algebraic operations follow the same conventions and order as arithmetic operations; use index notation and the index laws

Know that algebraic operations follow the same conventions and order as arithmetic operations.

Begin to generalise from arithmetic that multiplication and division have precedence over addition and subtraction. For example:

- In the expression $2 + 5a$, the multiplication is to be performed first.

Recognise that calculators that allow a whole calculation to be displayed apply the conventions of arithmetic, so $2 + 3 \times 4$ will be evaluated as 14, because 3×4 is evaluated first; other calculators may give 20 unless brackets are used: $2 + (3 \times 4)$.

Know that the commutative and associative laws apply to algebraic expressions as they do to arithmetic expressions, so:

- $2 + 3 = 3 + 2$ — $a + b = b + a$
- $2 \times 3 = 3 \times 2$ — $a \times b = b \times a$ or $ab = ba$
- $2 + (3 + 4) = (2 + 3) + 4$ — $a + (b + c) = (a + b) + c$
- $2 \times (3 \times 4) = (2 \times 3) \times 4$ — $a \times (b \times c) = (a \times b) \times c$ or $a(bc) = (ab)c$

Link to arithmetic operations (pages 84–5).

Inverses

Understand addition and subtraction as the inverse of each other, and multiplication and division as the inverse of each other. Generalise from arithmetic that:

- $a + b = 5$ implies $b + a = 5$, using the commutative law, and the corresponding inverse relationships $5 - b = a$ and $5 - a = b$.
- Similarly, $a \times b = 24$ implies that $b \times a = 24$, $b = 24/a$ and $a = 24/b$.

Verify by substituting suitable sets of numbers.

Begin to apply inverse operations when two successive operations are involved. For example:

- The inverse of multiplying by 6 and adding 4 is subtracting 4 and dividing by 6, i.e.
  if $a \times 6 + 4 = 34$, then $(34 - 4)/6 = a$.

Alternatively:

- I think of a number, multiply by 6 and add on 4.
  The answer is 34.
  What was the original number?

Use a **calculator** to verify, pressing the equals key between each operation. For example, starting with 5:

$\boxed{5}\ \boxed{\times}\ \boxed{6}\ \boxed{=}\ \boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{-}\ \boxed{4}\ \boxed{=}\ \boxed{\div}\ \boxed{6}\ \boxed{=}$

should result in the same number as originally entered, i.e. 5. Check and explain what happens when the = key is not pressed at the various stages.

Link to arithmetic operations (pages 84–5).

As outcomes, Year 8 pupils should, for example:

Recognise that algebraic operations follow the same conventions and order as arithmetic operations.
Know that contents of brackets are evaluated first, and that multiplication and division are carried out before addition and subtraction. For example:

- In $7 - 5s$, the multiplication is performed first.
- In $6 - s^2$, the square is evaluated first.
- In $3(x - 2)$, the expression in the brackets is evaluated first.

Use index notation for small positive integer powers.
Know that expressions involving repeated multiplication of the same number, such as:

$n \times n \qquad n \times n \times n \qquad n \times n \times n \times n$

are written as n^2, n^3 and n^4, and are referred to as *n squared*, *n cubed*, *n to the power of 4*, etc.
Know why the terms *squared* and *cubed* are used for *to the power of 2* and *to the power of 3*.

Understand the different meanings of expressions such as:

$2n$ and $n^2 \qquad 3n$ and n^3

Simplify expressions such as:

$2x^2 + 3x^2 \qquad n^2 \times n^3 \qquad p^3 \div p^2$

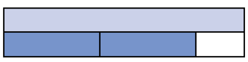
Understand and use inverse operations.
Recognise that any one of the equations:

- $a + b = c$, $b + a = c$, $c - a = b$ and $c - b = a$
- $ab = c$, $ba = c$, $b = c/a$ and $a = c/b$

implies each of the other three in the same set, as can be verified by substituting suitable sets of numbers into the equations.

Use coloured rods, e.g. white (1 unit), red (2 units) and yellow (5 units), to express relationships such as:

- $y = 2r + w \qquad w = y - 2r \qquad r = (y - w)/2$



Apply inverse operations when two successive operations are involved. For example:

- The inverse of dividing by 4 and subtracting 7 is adding 7 and multiplying by 4, i.e.
  if $m/4 - 7 = n$, then $m = 4(n + 7)$.

Use a **spreadsheet** to verify this, entering different numbers in column A, including negative numbers and decimals. If column C always equals column A, the inverse is probably correctly expressed.

	A	B	C
1	Number	=A1/4–7	=(B1+7)*4
2		=A2/4–7	=(B2+7)*4
3		=A3/4–7	=(B3+7)*4
4		=A4/4–7	=(B4+7)*4
5		=A5/4–7	=(B5+7)*4

	A	B	C
1	1	–6.75	1
2	2	–6.5	2
3	3	–6.25	3
4	4	–6	4
5	5	–5.75	5

Link to arithmetic operations (pages 84–5).

As outcomes, Year 9 pupils should, for example:

Apply simple instances of the index laws for multiplication and division of small integer powers.
For example:

- $n^2 \times n^3 = n^{2+3} = n^5$
- $p^3 \div p^2 = p^{3-2} = p$

See page 59.

Know and use the general forms of the index laws for multiplication and division of positive integer powers.

$p^a \times p^b = p^{a+b} \qquad p^a \div p^b = p^{a-b} \qquad (p^a)^b = p^{ab}$

Begin to extend understanding of index notation to negative and fractional powers and recognise that the index laws can be applied to these as well.

See page 59.

Pupils should be taught to:

Simplify or transform algebraic expressions

As outcomes, Year 7 pupils should, for example:

Simplify linear expressions by collecting like terms; begin to multiply a single term over a bracket.

Understand that partitioning a number helps to break a multiplication into a series of steps. For example:

- By partitioning 38, 38×7 becomes
  $(30 + 8) \times 7 = 30 \times 7 + 8 \times 7$

	30	8
7	210	56

  Generalise, from this and similar examples, to:
  $(a + b) \times c = (a \times c) + (b \times c)$
  or $ac + bc$

	a	b
c	$a \times c$	$b \times c$

Link to written methods for multiplication (pages 104–5).

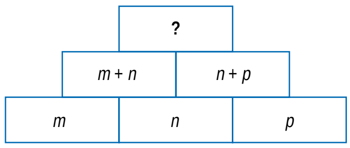
Recognise that letters stand for numbers in problems. For example:

- Simplify expressions such as:

  a. $a + a + a = 3a$
  b. $b + 2b + b = 4b$
  c. $x + 6 + 2x = 3x + 6$
  d. $3n + 2n = 5n$
  e. $3(n + 2) = 3n + 6$

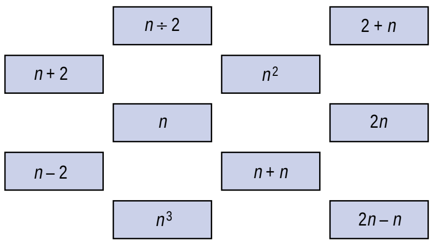
  and $a/a = 1$, $2a/a = 2$, … and $4a/2 = 2a$, $6a/2 = 3a$, etc.

- The number in each cell is the result of adding the numbers in the two cells beneath it.



  Write an expression for the number in the top cell.
  Write your expression as simply as possible.

- Here are some algebra cards.



  a. Which card will always give the same answer as $n/2$?
  b. Which card will always give the same answer as $n \times n$?
  c. Two cards will always give the same answer as $2 \times n$. Which cards are they?
  d. When the expressions on three of the cards are added together they will always have the same answer as $5n$. Which cards are they?
  e. Write a new card that will always give the same answer as $3n + 2n$.

- Draw some shapes that have a perimeter of $6x + 12$.

- The answer is $2a + 5b$. What was the question?

As outcomes, Year 8 pupils should, for example:

Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket.

Understand the application of the distributive law to arithmetic calculations such as:

- $7 \times 36 = 7(30 + 6) = 7 \times 30 + 7 \times 6$
- $7 \times 49 = 7(50 - 1) = 7 \times 50 - 7 \times 1$

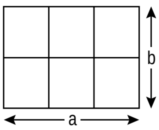
Know and use the distributive law for multiplication:

- over addition, namely $a(b + c) = ab + ac$
- over subtraction, namely $a(b - c) = ab - ac$

Recognise that letters stand for numbers in problems. For example:

- Simplify these expressions:
  a. $3a + 2b + 2a - b$
  b. $4x + 7 + 3x - 3 - x$
  c. $3(x + 5)$
  d. $12 - (n - 3)$
  e. $m(n - p)$
  f. $4(a + 2b) - 2(2a + b)$

- Write different equivalent expressions for the total length of the lines in this diagram.

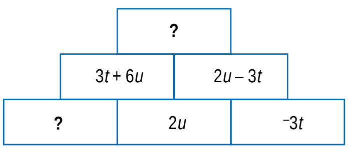


  Simplify each expression as far as possible.
  What did you discover?

- In a magic square the sum of the expressions in each row, column and diagonal is the same.
  Show that this square is a magic square.

$m - p$	$m + p - q$	$m + q$
$m + p + q$	m	$m - p - q$
$m - q$	$m - p + q$	$m + p$

- The number in each cell is made by adding the numbers in the two cells beneath it.
  Fill in the missing expressions.
  Write each expression as simply as possible.



As outcomes, Year 9 pupils should, for example:

Simplify or transform expressions by taking out single-term common factors.

Continue to use the distributive law to multiply a single term over a bracket.

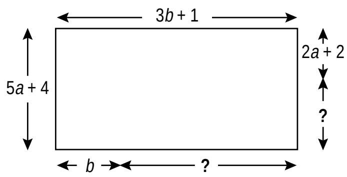
Extend to taking out single-term common factors.

Recognise that letters stand for numbers in problems. For example:

- Simplify these expressions:
  $3(x - 2) - 2(4 - 3x)$
  $(n + 1)^2 - (n + 1) + 1$

- Factorise:
  $3a + 6b = 3(a + 2b)$
  $x^3 + x^2 + 2x = x(x^2 + x + 2)$

- Write an expression for each missing length in this rectangle. Write each expression as simply as possible.



- The area of a rectangle is $2x^2 + 4x$.
  Suggest suitable lengths for its sides.
  What if the perimeter of a rectangle is $2x^2 + 4x$?

- Prove that the sum of three consecutive integers is always a multiple of 3.
  Let the integers be n, $n + 1$ and $n + 2$.
  Sum $= n + (n + 1) + (n + 2)$
  $= 3n + 3$
  $= 3(n + 1)$, which is a multiple of 3.

- Think of a number, multiply by 3, add 15, divide by 3, subtract 5. Record your answer.
  Try other starting numbers. What do you notice?
  Use algebra to prove the result.

- *What is the smallest value you can get for $x^2 - x$ if x is an integer?*
  What is the smallest value if x does not have to be an integer?
  *Use a **spreadsheet** to help.*

- *Prove that the value of $x^3 - x + 9$ is divisible by 3 for any integer value of x.*

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Simplify or transform algebraic expressions (continued)	

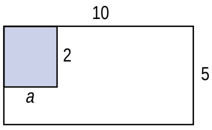
As outcomes, Year 8 pupils should, for example:

Explore general algebraic relationships.

For example:

- By dividing this shape into rectangles in different ways, form different equivalent expressions for the unshaded area, for example:

  $50 - 2a$
  $2(10 - a) + 30$
  $3a + 5(10 - a)$



  Then multiply out and simplify the expressions to show in a different way that they are equivalent.

- Use a **spreadsheet** to verify that $2a + 2b$ has the same value as $2(a + b)$ for any values of a and b, e.g. set up the expressions in columns C and D.

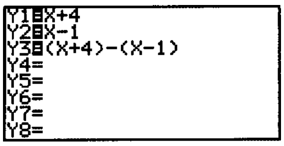
	A	B	C	D
1			=2*A1+2*B1	=2*(A1+B1)

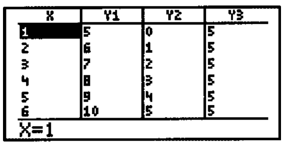
	A	B	C	D
1	3	7	20	20

  Repeatedly enter a different pair of numbers in columns A and B. Try positive and negative numbers, and whole numbers and decimals.

  Recognise that if equivalent expressions have been entered in C1 and D1, then columns C and D will always show the same number.

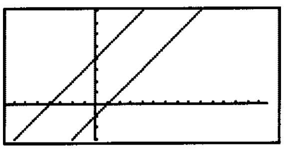
- Use a **graphical calculator** to verify that $(x + 4) - (x - 1) = 5$, for any value of x.





  Use the table to confirm that $4 - {}^{-}1 = 5$.

  Draw the graphs of the two straight lines $y = x + 4$ and $y = x - 1$.



  Confirm that the vertical distance beween the two lines is always 5, i.e. $(x + 4) - (x - 1) = 5$.

As outcomes, Year 9 pupils should, for example:

Add simple algebraic fractions.

Generalise from arithmetic that:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

- This is what a pupil wrote. Show that the pupil was wrong.

  For all numbers t and w

  $$\frac{1}{t} + \frac{1}{w} = \frac{2}{t + w}$$

Square a linear expression, and expand and simplify the product of two linear expressions of the form $x \pm n$.

Apply the distributive law to calculations such as:

- $53 \times 37 = (50 + 3)(30 + 7)$
  $= 50 \times 30 + 3 \times 30 + 50 \times 7 + 3 \times 7$
- $57 \times 29 = (50 + 7)(30 - 1)$
  $= 50 \times 30 + 7 \times 30 - 50 \times 1 - 7 \times 1$

Derive and use identities for the product of two linear expressions of the form $(a + b)(c \pm d)$:

- $(a + b)(c + d) \equiv ac + bc + ad + bd$
- $(a + b)(a + b) \equiv a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$
- $(a + b)(c - d) \equiv ac + bc - ad - bd$
- $(a + b)(a - b) \equiv a^2 - ab + ab - b^2 = a^2 - b^2$

	a	b	
c	ac	bc	ac + bc
d	ad	bd	ad + bd
			ac + bc + ad + bd

For example:

- *Multiply out and simplify:*

  $(p - q)^2 = (p - q)(p - q)$
  $= p^2 - pq - pq + q^2$
  $= p^2 - 2pq + q^2$

  $(3x + 2)^2 = (3x + 2)(3x + 2)$
  $= 9x^2 + 6x + 6x + 4$
  $= 9x^2 + 12x + 4$

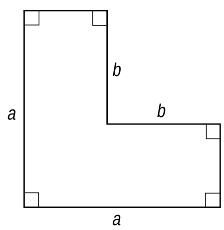
  $(x + 4)(x - 3) = x^2 + 4x - 3x - 12$
  $= x^2 + x - 12$

Use geometric arguments to prove these results. For example:

- *Multiply out and simplify these expressions to show that they are equivalent.*

  $a^2 - b^2$
  $a(a - b) + b(a - b)$
  $2b(a - b) + (a - b)(a - b)$
  $(a - b)(a + b)$

  By formulating the area of this shape in different ways, use geometric arguments to show that the expressions are equivalent.

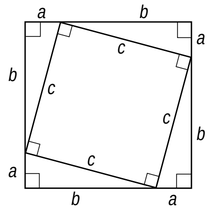


Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Simplify or transform algebraic expressions (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

- *Four identical right-angled triangles with hypotenuse of length c are placed as shown.*

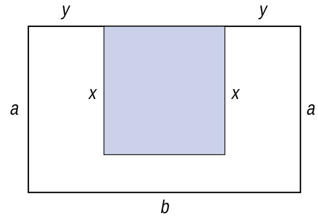
  

  Show that the inner shape formed is a square of side c, and the outer shape formed is a square of side a + b.

  Use the diagram to deduce a proof of Pythagoras' theorem:

  $c^2 = (a + b)^2 - 4(\frac{1}{2}ab)$
  $= (a^2 + 2ab + b^2) - 2ab$
  $= a^2 + b^2$

- *Imagine a room with an area of carpet (shaded).*

  By dividing the room into rectangles in different ways, find different equivalent expressions for the floor area with no carpet, for example:

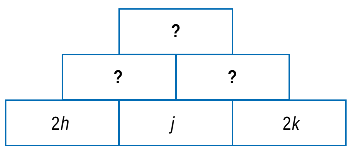
  $A = 2xy + b(a - x)$
  $A = 2ay + (a - x)(b - 2y)$

  Multiply out the expressions to confirm that they are equivalent.

Solve problems such as:

- *Prove that the product of two odd numbers is always odd.*

- *In this diagram, h, j and k can be any integers. The missing number in each cell is found by adding the two numbers beneath it. Prove that the number in the top cell will always be even.*

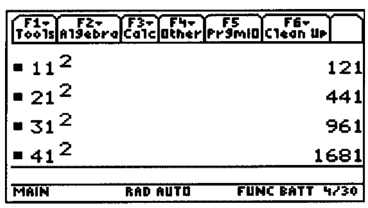
  

  What if j is replaced by j + 1?
  What if 2h is replaced by h?

- *Show that:*

  $(n + 1)^2 = n^2 + 2n + 1$

  Use this result to calculate 91^2, 801^2.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Construct and solve linear equations, selecting an appropriate method

Use, read and write, spelling correctly:
equation, solution, unknown, solve, verify, prove, therefore (∴).

Construct and solve simple linear equations with integer coefficients, the unknown on one side only.

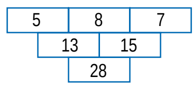
Choose a suitable unknown and form expressions leading to an equation. Solve the equation by using inverse operations or other mental or written methods.

For example:

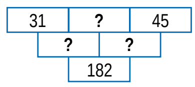
- I think of a number, subtract 7 and the answer is 16.
  What is my number?
  Let n be the number.
  $n - 7 = 16$
  $\therefore\ n = 16 + 7 = 23$

- A stack of 50 sheets of card is 12 cm high.
  How thick is one sheet of card?
  Let d cm be the thickness of each sheet.
  $50d = 12$
  $\therefore\ d = \frac{12}{50} = \frac{24}{100} = 0.24$
  The thickness of each sheet is 0.24 cm.

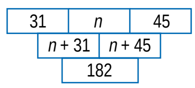
- In this diagram, the number in each cell is formed by adding the two numbers above it.



  What are the missing numbers in this diagram?



  Let n be the number in the top centre cell. Form the first row and the subsequent row. It follows that:



  $n + 31 + n + 45 = 182$
  $\therefore\ 2n = 106$
  $n = 53$

  What if the top three numbers are swapped around?
  What if you start with four numbers?

- I think of a number, multiply it by 6 and add 1.
  The answer is 37. What is my number?

- There are 26 biscuits altogether on two plates.
  The second plate has 8 fewer biscuits than the first plate.
  How many biscuits are there on each plate?

- Find the angle a in a triangle with angles a, $a + 10$, $a + 20$.

- Solve these equations:

a. $a + 5 = 12$	c. $7h - 3 = 20$	e. $2c + 3 = 19$
b. $3m = 18$	d. $7 = 5 + 2z$	f. $6 = 2p - 8$

Check solutions by substituting into the original equation.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
linear equation…

Consolidate forming and solving linear equations with an unknown on one side.

Choose a suitable unknown and form expressions leading to an equation. Solve the equation by removing brackets, where appropriate, collecting like terms and using inverse operations.

For example:

- There are 376 stones in three piles. The second pile has 24 more stones than the first pile. The third pile has twice as many stones as the second. How many stones are there in each pile?

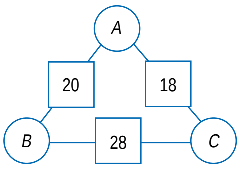
  Let s stand for the number in the first pile.

Pile 1	Pile 2	Pile 3	Total
s	$s + 24$	$2(s + 24)$	376

  $$\begin{aligned} s + (s + 24) + 2(s + 24) &= 376 \\ \therefore\ s + s + 24 + 2s + 48 &= 376 \\ 4s + 72 &= 376 \\ 4s &= 376 - 72 = 304 \\ s &= 76 \end{aligned}$$

- In an arithmagon, the number in a square is the sum of the numbers in the two circles on either side of it.

  In this triangular arithmagon, what could the numbers *A*, *B* and *C* be?



  Let x stand for the number in the top circle. Form expressions for the numbers in the other circles, (20 – x) and (18 – x). Then form an equation in x and solve it.

  $$\begin{aligned} (20 - x) + (18 - x) &= 28 \\ \therefore\ 38 - 2x &= 28 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

  So $A = 5,\ B = 15,\ C = 13.$

- On Dwain's next birthday, half of his age will be 16. How old is Dwain now?

- Solve these equations:

  a. $5x = 7$
  b. $3 = \frac{12}{n}$
  c. $2(p + 5) = 24$
  d. $2.4z + 5.9 = 14.3$
  e. $4(b - 1) + 5(b + 1) = 100$

Check solutions by substituting into the original equation.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
inequality, region… and, or…

Construct and solve linear equations with negative signs anywhere in the equation, negative solution…

Solve linear equations using inverse operations, by transforming both sides in the same way or by other methods.

For example:

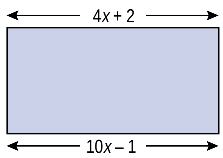
- Compare different ways of solving 'think of a number' problems and decide which would be more efficient – retaining brackets and using inverse operations, or removing brackets first.
  For example:

  I think of a number, add 3, multiply by 4, add 7, divide by 9, then multiply by 15.
  The final answer is 105.
  What was the number that I thought of?

- Jack, Jo and Jim are sailors.
  They were shipwrecked on an island with a monkey and a crate of 185 bananas.
  Jack ate 5 more bananas than Jim.
  Jo ate 3 more bananas than Jim.
  The monkey ate 6 bananas.
  How many bananas did each sailor eat?

- The length of a rectangle is three times its width.
  Its perimeter is 24 centimetres.
  Find its area.

- The area of this rectangle is 10 cm².



  Calculate the value of *x* and use it to find the length and width of the rectangle.

- In ΔABC, ∠B is three quarters of ∠A, and ∠C is one half of ∠A. Find all the angles of the triangle.

- Solve these equations:

  a. $3c - 7 = {}^{-}13$
  b. $1.7m^2 = 10.625$
  c. $4(z + 5) = 8$
  d. $4(b - 1) - 5(b + 1) = 0$
  e. $\frac{12}{(x + 1)} = \frac{21}{(x + 4)}$

Check solutions by substituting into the original equation.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

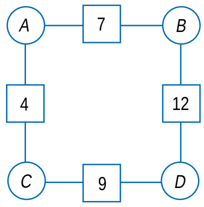
Construct and solve linear equations, selecting an appropriate method (continued)

Explore ways of constructing simple equations to express relationships, and begin to recognise equivalent statements. For example:

- In an arithmagon, the number in a square is the sum of the numbers in the two circles either side of it.

  In this square arithmagon, what could the numbers *A*, *B*, *C* and *D* be?

  Can you find any relationships between *A*, *B*, *C* and *D*?



Some results

A	B	C	D
3	4	1	8
2	5	2	7
1	6	3	6
4	3	0	9
0	7	4	5

Some relationships

$A + B = 7$
$C + D = 9$
$A + B + C + D = 16$
$D - A = 5$
$B - C = 3$
$B = C + 3$

Recognise that statements such as $B - C = 3$ and $B = C + 3$ express the same relationship in different ways.

As outcomes, Year 8 pupils should, for example:

Explore alternative ways of solving simple equations, e.g. deciding whether or not to remove brackets first. For example:

- $$\begin{aligned} 2(x+5) &= 36 \\ x+5 &= 18 \\ x &= 13 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 2(x+5) &= 36 \\ 2x+10 &= 36 \\ 2x &= 26 \\ x &= 13 \end{aligned}$$

Begin to understand that an equation can be thought of as a balance where, provided the same operation is performed on both sides, the resulting equation remains true. For example:

- Start with a true statement, such as:
  $52 - 7 = 41 + 4$
  Make the same change to both sides, e.g. subtract 4:
  $52 - 7 - 4 = 41$
  Check that this statement is true.

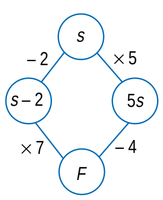
  Then start with a simple equation, such as:
  $$\begin{array}{lrcl} & y &=& x+4 \\ \text{add 3} & y+3 &=& x+7 \\ \text{double} & 2(y+3) &=& 2(x+7) \\ \text{subtract } d & 2(y+3)-d &=& 2(x+7)-d \end{array}$$
  Check that the resulting equation is true by substituting numbers which fit the original, e.g. $x = 1$, $y = 5$.

Form linear equations (unknown on both sides) and solve them by transforming both sides in the same way. Begin to recognise what transformations are needed and in what order. For example:

- Jill and Ben each have the same number of pens. Jill has 3 full boxes of pens and 2 loose pens. Ben has 2 full boxes of pens and 14 loose pens. How many pens are there in a full box?
  $3n + 2 = 2n + 14$

- In the two-way flow diagram, find the starting number s that has to be entered, so that you reach the same finishing number F, whichever route is followed.

  $$\begin{aligned} 7(s-2) &= 5s-4 \\ \therefore\ 7s-14 &= 5s-4 \\ 7s &= 5s-4+14 \\ 7s &= 5s+10 \\ 2s &= 10 \\ s &= 5 \end{aligned}$$



- Solve these equations:
  a. $3x + 2 = 2x + 5$
  b. $5z - 7 = 13 - 3z$
  c. $4(n + 3) = 6(n - 1)$

Check solutions by substituting into the original equation.

As outcomes, Year 9 pupils should, for example:

Form linear equations (unknown on both sides) and solve them by transforming both sides in the same way. For example:

- Multiplying a number by 2 and then adding 5 gives the same answer as subtracting the number from 23. What is the number?

- Françoise and Jeanette have 250 euros between them. Jeanette gave Françoise 50 euros. Françoise now has four times as many euros as Jeanette. How many euros has Françoise?

- The sum of the ages of a mother and her daughter is 46. In three years' time the mother will be three times as old as her daughter is then. How old is the daughter now?

- Solve these equations:
  a. $7(s + 3) = 45 - 3(12 - s)$
  b. $3(2a - 1) = 5(4a - 1) - 4(3a - 2)$
  c. $2(m - 0.3) - 3(m - 1.3) = 4(3m + 3.1)$
  d. $\frac{3}{4}(c - 1) = \frac{1}{2}(5c - 3)$
  e. $\dfrac{x-3}{2} = \dfrac{x-2}{3}$

Check solutions by substituting into the original equation.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Solve a pair of simultaneous linear equations	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Solve a pair of simultaneous linear equations by eliminating one variable.

*Know that **simultaneous equations** are true at the same time and are satisfied by the same values of the unknowns involved, and that linear simultaneous equations may be solved in a variety of ways.*

Substitute from one equation into another.
For example:

- *x and y satisfy the equation $5x + y = 49$.*
  They also satisfy the equation $y = 2x$.
  Find x and y.
  Write down another equation satisfied by x and y.

  Method
  From the second equation, $y = 2x$.
  Substituting into the first equation gives
  $5x + 2x = 49$
  So $x = 7$ and $y = 14$.
  Other equations might be $x + y = 21$, $y = x + 7$.

- *Solve the equations:*
  a. $5p - q = 30$, $q = 3p$ b. $3x - 5y = 22$, $x = 3y + 2$

Extend the substitution method to examples where one equation must be rearranged before the substitution can be made. For example:

- *Solve the equations:* $x - 2y = 5$, $2x + 5y = 100$
  From the first equation, $x = 5 + 2y$.
  Substituting into the second equation gives
  $2(5 + 2y) + 5y = 100$

Add or subtract equations. *For example:*

- *Solve the equations:*
  $x + 3y = 56$
  $x + 6y = 101$

  Method $x + 3y = 56$
  $x + 6y = 101$
  Compare the two equations and deduce that $3y = 45$ so $y = 15$.
  Substituting into the first equation gives $x = 11$.

- *Solve the equations:*
  a. $4x + y = 44$, $x + y = 20$ b. $5x + y = 17$, $5x - y = 3$

Extend to adding or subtracting equations in order to eliminate one variable. For example:

- $2x + y = 17$ *Multiply by 2:* $4x + 2y = 34$
  $3x + 2y = 28$ *Subtract:* $3x + 2y = 28$
  $x = 6$

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Solve a pair of simultaneous linear equations (continued)	

As outcomes, Year 8 pupils should, for example:

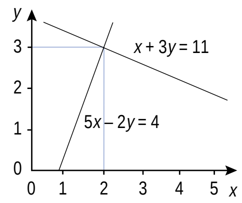
As outcomes, Year 9 pupils should, for example:

Use a graphical method and check algebraically.

Use pencil and paper or a ***graph plotter*** *or* ***graphical calculator*** *to draw the graph of each equation.*

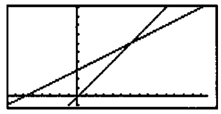
- *Solve:*

  $x + 3y = 11$
  $5x - 2y = 4$



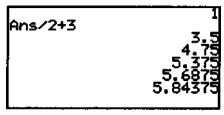
The intersection of the two lines, the point (2, 3), gives an approximate solution to the equations.

- $y = x$
  $y = \frac{x}{2} + 3$



Link the solution $x = 6$, $y = 6$ of this pair of equations to finding the limit of the sequence ‘divide by 2, add 3’, starting with 1, represented as

$x \rightarrow \frac{x}{2} + 3$



Recognise that:

- *The point at which the graphs of two equations intersect lies on both lines; its coordinates give the simultaneous solution of the two equations.*
- *Equations such as $x + y = 7$ and $3x + 3y = 21$ have an infinite number of solutions, since they are represented by the same graph.*
- *Equations such as $y = 4x + 5$ and $y = 4x + 10$ have no simultaneous solution, since their graphs are parallel lines which never meet.*
- *If the graphs of three (or more) equations in two unknowns pass through a common point then the equations have a common solution, given by the coordinates of the point. If the graphs are not coincident then the equations they represent have no common solution.*

Form and solve linear simultaneous equations to solve problems. For example:

- *In five years’ time, Ravi’s father will be twice as old as Ravi. In 13 years’ time, the sum of their ages will be 100. How old is Ravi now?*
- *A nursery was asked to plant a number of trees in a number of days. If the nursery plants 240 trees per day, then 400 fewer trees than planned will be planted. If it plants 280 trees daily, it will plant 200 more than planned.*
  How many trees should the nursery plant?
  How many days will it take?
- *Find x and y when $x + 3y = 10$ and $2x + y = 5$.*
  Invent a problem that could give rise to these two simultaneous equations.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Solve linear inequalities in one variable, and represent the solution set on a number line; begin to solve inequalities in two variables	

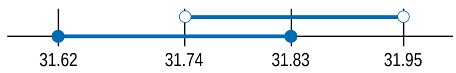
As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Solve linear inequalities in one variable; represent the solution set on a number line. For example:

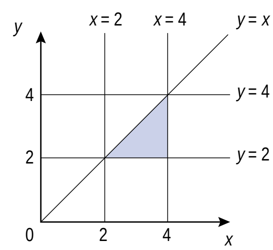
- *An integer n satisfies $^{-}8 < 2n \le 10$.*
  List all the possible values of n.
  ($n = {}^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3, 4, 5$)
- *Find all the possible integer values of q that satisfy $2q < 17$ and $3q > 7$.*
- *Show the solution of the inequality $^{-}2 < z \le 3$ on a number line, for example:*

- *Solve these inequalities, marking the solution set on a number line:*
  a. $3n + 4 < 17$ and $n > 2$
  b. $2(x - 5) \le 0$ and $x > {}^{-}2$
- *The variable y satisfies each of these inequalities:*
  $5 - 2y \le 13$
  $4y + 6 \le 10$
  Mark the solution set for y on a number line.
- *The variable b satisfies each of these inequalities:*
  $31.62 \le b \le 31.83$ and $31.95 > b > 31.74$
  Mark the solution set for b on a number line.



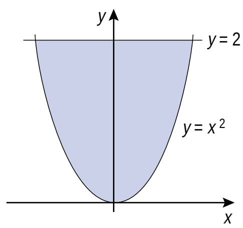
Begin to solve inequalities in two variables.
For example:

- *This pattern is formed by straight-line graphs of equations in the first quadrant.*



Write three inequalities to describe fully the shaded region.

- *The shaded region is bounded by the line $y = 2$ and the curve $y = x^2$.*



Circle two inequalities that fully describe the shaded region.

$y < x^2$	$x \le 0$	$y \le 2$	$y < 0$
$y \ge x^2$	$x > 0$	$y > 2$	$y \ge 0$

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Use systematic trial and improvement methods and ICT tools to find solutions, or approximate solutions, to non-linear equations	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Use algebraic methods to solve simple non-linear equations. For example:

- Solve these equations exactly.
  Each has two solutions.

  a. $c^2 + 24 = 60$
  b. $\frac{9}{y + 2} = y + 2$
  c. $x^2 - 5 = 220$
  d. $3 = \frac{12}{x^2}$

Use systematic trial and improvement methods and ICT tools to find solutions, or approximate solutions, to equations.

Use a **calculator** to answer questions such as:

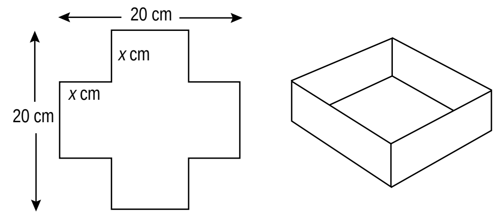
- Solve these equations, giving answers correct to two decimal places:
  a. $s^3 = 30$
  b. $a^3 + a = 50$
  c. $z^3 - z = 70$
  d. $5.5p^3 = 9.504$

- The product of three consecutive odd numbers is 205 143. Find the numbers.

- A cuboid has a square cross-section (side x cm), height 20 cm and total surface area 800 cm^2. Form and solve an equation in x, giving the answer correct to one decimal place.

Set up an equation for a problem and find an approximate solution using a **spreadsheet** or a **graph plotter**. For example:

- A small open box is made by starting with a sheet of metal 20 cm by 20 cm, cutting squares from each corner and folding pieces up to make the sides.



  The box is to have a capacity of 450 cm^3 to its rim. Use a **spreadsheet** to find what size of square, to the nearest millimetre, should be cut from the corners.

  Use a **graphical calculator** or **graph plotting program** to find what size of square should be cut from the corners to make a box with the maximum possible volume.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Use systematic trial and improvement methods and ICT tools to find solutions, or approximate solutions, to non-linear equations (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

- The length of one side of a rectangle is y.
  This equation shows the area of the rectangle.

  $$y(y + 2) = 67.89$$

  Find the value of y. Show your working.

  You may find this table helpful.

y	$y+2$	$y(y+2)$	
8	10	80	too large

- Complete the table below for values of x and y for the equation $y = x^3 - x - 10$.

x	0	1	2	3	4	5	6
y							

  The value of y is 0 for a value of x between 2 and 3.
  Find the value of x, to 1 decimal place, that gives the value of y closest to 0.

  Use trial and improvement.

  Using a **spreadsheet,** the solution lies between 2 and 3.

	A	B
1	x	x³ – x – 10
2	2	=A2*A2*A2–A2–10
3	=A2+0.1	=A3*A3*A3–A3–10
4	=A3+0.1	=A4*A4*A4–A4–10
5	=A4+0.1	=A5*A5*A5–A5–10
6	=A5+0.1	=A6*A6*A6–A6–10
7	=A6+0.1	=A7*A7*A7–A7–10
8	=A7+0.1	=A8*A8*A8–A8–10
9	=A8+0.1	=A9*A9*A9–A9–10
10	=A9+0.1	=A10*A10*A10–A10–10
11	=A10+0.1	=A11*A11*A11–A11–10
12	=A11+0.1	=A12*A12*A12–A12–10

	A	B
1	x	x³ – x – 10
2	2	–4
3	2.1	–2.839
4	2.2	–1.552
5	2.3	–0.133
6	2.4	1.424
7	2.5	3.125
8	2.6	4.976
9	2.7	6.983
10	2.8	9.152
11	2.9	11.489
12	3	14

  The table shows that y has the value 0 for a value of x between 2.3 and 2.4.

  Adjust column A to examine values of y for these values of x. This confirms that $x = 2.31$ gives the value of y closest to 0, that is, $x = 2.3$ to 1 d.p.

	A	B
1	x	x³ – x – 10
2	2.3	–0.133
3	2.31	0.016391
4	2.32	0.167168
5	2.33	0.319337
6	2.34	0.472904
7	2.35	0.627875
8	2.36	0.784256
9	2.37	0.942053
10	2.38	1.101272
11	2.39	1.261919
12	2.4	1.424

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Set up and use equations to solve word and other problems involving direct proportion	

As outcomes, Year 8 pupils should, for example:

Begin to use graphs and set up equations to solve simple problems involving direct proportion.

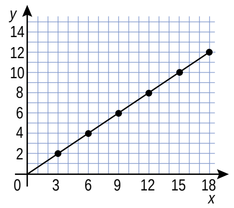
Discuss practical examples of direct proportion, such as:

- the number of euros you can buy for any amount in pounds sterling (no commission fee);
- the number of kilometres equal to a given number of miles, assuming 8 km to every 5 miles;
- the cost of tennis balls, originally at £3 each, offered at £2 each in a sale. For example, generate sets of proportional pairs by multiplying and test successive ratios to see if they are equal:

Original price $£3 \xrightarrow{\times 2} £6 \xrightarrow{\times 3} £9 \xrightarrow{\times 4} £12...$

Sale price $£2 \xrightarrow{\times 2} £4 \xrightarrow{\times 3} £6 \xrightarrow{\times 4} £8...$

Check by drawing a graph, using pencil and paper or **ICT**.

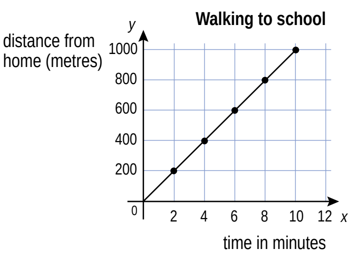


Observe that the points lie in a straight line through the origin, and that the sale price (y) is related to the original price (x) by the formula:

$y = \frac{2}{3} x$

Discuss the distance travelled in a given time, assuming constant speed. For example:

- Consider a walking speed of 200 metres every 2 minutes.



Generate (distance, time) pairs.
Observe that distance/time is constant.
Use this relationship to find the distance walked in:
a. 12 minutes, b. 7 minutes.

Link to ratio and proportional reasoning in number (pages 78–81).

As outcomes, Year 9 pupils should, for example:

Solve problems involving proportion using algebraic methods, relating algebraic solutions to graphical representations of the equations. For example:

Understand direct proportion as equality of ratios.
If variables x and y take these values:

x	1	2	3	4	5	6	...
y	3.5	7	10.5	14	17.5	21	...

- the ratios of corresponding values of y and x are equal:

$$\frac{y}{x} = \frac{3.5}{1} = \frac{7}{2} = \frac{10.5}{3} = \frac{14}{4} = \frac{17.5}{5} = \frac{21}{6} = \ldots = 3.5$$

- y is said to be directly proportional to x;
- the relationship between the variables is expressed by $y = 3.5x$;
- the graph of y against x will be a straight line through the origin.

Check whether two sets of values are in direct proportion by comparing corresponding ratios.

Use algebraic methods to solve problems such as:

- Green paint is made by mixing 11 parts of blue paint with 4 parts of yellow paint.
  How many litres of blue paint would be needed to mix with 70 litres of yellow paint?

  Algebraic method
  Let b be the number of litres of blue paint needed.

  $$\frac{\text{blue}}{\text{yellow}} \qquad \frac{b}{70} = \frac{11}{4}$$

  How many litres of blue paint would be needed to make up 100 tins of green paint?

  $$\frac{\text{blue}}{\text{green}} \qquad \frac{b}{100} = \frac{11}{15}$$

Link proportionality to work in science.
Appreciate that where data are obtained from experimental measurements:

- limitations in measuring mean that exactly equal ratios are unlikely and plotted points may only approximate to a straight line;
- significant deviations of individual observations often indicate experimental error, such as misreading a value.

Link to ratio and proportion in number (pages 78–81), gradients of lines of the form $y = mx$ (pages 166–7), graphs of real situations (pages 172–7), lines of best fit (pages 266–7), and scale factor (pages 212–15).

Pupils should be taught to:

Use formulae from mathematics and other subjects

As outcomes, Year 7 pupils should, for example:

Substitute positive integers into simple linear expressions.

For example:

- Substitute positive integer values into:

$x + y - z$	$3(x + y)$	$20/x$
$9y - x$	$2(8 - x)$	$x/2 - 6$

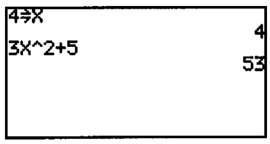
- Check that these statements are true for particular values by substituting the values into each expression and its simplification.

$a + a + a = 3a$	$3n + 2n = 5n$	$6n/2 = 3n$
$b + 2b + b = 4b$	$3(n + 2) = 3n + 6$	

- Use a **spreadsheet**.
  Enter a formula such as 3A + B in column C. Find six different ways of putting different numbers in columns A and B to produce, say, 56 in column C.

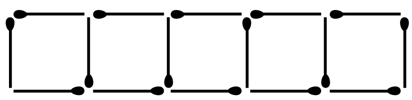
  Try other formulae in column C.

- Use a **graphical calculator** to substitute numbers in expressions such as $3x^2 + 5$.



- If n is an integer, the expressions
  $2n + 1$, $2n + 3$, $2n + 5$, $2n + 7$ and $2n + 9$
  represent a set of five consecutive odd numbers.
  Explain why.
  What value of n would produce the set of odd numbers 9, 11, 13, 15, 17?

- The expression $3s + 1$ gives the number of matches needed to make a row of s squares.



  How many matches are needed to make a row of 13 squares?

As outcomes, Year 8 pupils should, for example:

Substitute positive and negative numbers into linear expressions and positive integers into simple expressions involving powers.

For example:

- Find the value of these expressions when $a = 4$.
  $3a^2 + 4$ $\quad$ $2a^3$

- Find the value of these expressions when $x = 2.5$.
  $4x + 3$ $\quad$ $2 - 3x$ $\quad$ $7(x - 1)$

- Find the value of y when $x = {}^{-}3$.
  $y = \dfrac{2x + 3}{x}$ $\quad$ $y = \dfrac{x - 1}{x + 1}$

- Use a short **computer program**. For example:

```
10 CLS
20 INPUT A
30 INPUT B
40 C = 6*A-2*B
50 PRINT C
60 PRINT
```

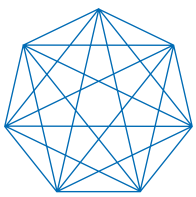
```
TO EXPRESSION :A :B
MAKE "C 6*:A-2*:B
PRINT :C
END
```

  Find different ways of inputting different values for A and B to print a particular value for C.
  Try different formulae.

- Use a **spreadsheet** to explore what happens when different values are substituted in an expression. For example:

	A	B	C
1	-3	7	=A1*A1-B1
2	-2	2	=A2*A2-B2
3	-1	8	=A3*A3-B3
4	0	-3	=A4*A4-B4
5	1	1	=A5*A5-B5
6	2	-4	=A6*A6-B6
7	3	5	=A7*A7-B7

- The number of diagonals in a polygon with n sides is given by the expression



  $$\frac{n(n-3)}{2}$$

  How many diagonals are there in a polygon with 20 sides?

- In this magic square, choose different values for m, p and q and substitute them.

$m - p$	$m + p - q$	$m + q$
$m + p + q$	m	$m - p - q$
$m - q$	$m - p + q$	$m + p$

  What values for m, p and q will give a magic square filled with the numbers 1 to 9?

As outcomes, Year 9 pupils should, for example:

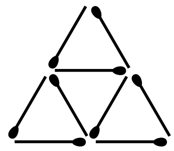
Substitute positive and negative numbers into linear expressions and expressions involving powers.

For example:

- Find the values of these expressions
  $3x^2 + 4$ $\quad$ $4x^3 - 2x$
  when $x = {}^{-}3$, and when $x = 0.1$.

- Find the values of a and b when $p = 10$.
  $a = \dfrac{3p^3}{2}$ $\quad$ $b = \dfrac{2p^2(p - 3)}{7p}$

- A triangle of matches is made like this.



  If the triangle has R rows, the number of matches needed is

  $$\tfrac{1}{2}(3R^2 + 3R)$$

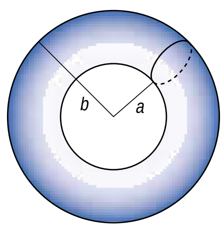
  How many matches are needed for a triangle with 17 rows?

- *A sphere with diameter 3.6 cm is made using clay. The volume of a sphere is*

  $$\frac{\pi d^3}{6},$$ *where d is the diameter.*

  Work out the volume of clay in the sphere. Give your answer to a sensible degree of accuracy.

  More clay is used to make this shape, a torus, with radii a = 4.5 and b = 7.5.



  Its volume is $\frac{1}{4}\pi^2(a + b)(b - a)^2$.
  Work out the volume of clay used.

- *Here are two formulae.*

  $$P = s + t + \frac{5\sqrt{(s^2 + t^2)}}{3} \qquad A = \tfrac{1}{2}st + \frac{(s^2 + t^2)}{9}$$

  Work out the values of P and A when s = 1.7 and t = 0.9.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use formulae from mathematics and other subjects (continued)

Explain the meaning of and substitute integers into formulae expressed in words, or partly in words, such as:

- number of days = 7 times the number of weeks
- cost = price of one item × number of items
- age in years = age in months ÷ 12
- pence = number of pounds × 100
- area of rectangle = length times width
- cost of petrol for a journey
  = cost per litre × number of litres used

Progress to substituting into formulae such as:

- conversion of centimetres c to metres m:
  $m = c \div 100$
- the area A of a rectangle of length l and width w:
  $A = lw$
- the perimeter P of a rectangle of length l and width w:
  $P = 2l + 2w$ or $P = 2(l + w)$
- the area A of a right-angled triangle with base b and height h:
  $A = \frac{1}{2}bh$

As outcomes, Year 8 pupils should, for example:

Explain the meaning of and substitute numbers into formulae such as:

- the volume V of a cuboid of length l, breadth b and height h:
  $V = lbh$
- the surface area S of a cuboid with width w, depth d and height h:
  $S = 2dw + 2dh + 2hw$

Answer questions such as:

- The voltage V in an electrical circuit, with current I and resistance R, is given by the formula:
  $V = IR$
  What is V when $I = 5$ and $R = 7$?
  What is R when $V = 42$ and $I = 3$?

In simple cases, find an unknown where it is not the subject of the formula and where an equation must be solved. For example:

- The formula for the change £C from £50 for d compact discs at £7 each is $C = 50 - 7d$.

  If $C = 15$, what is d?

- The formula for the perimeter P of a rectangle l by w is
  $P = 2(l + w)$.

  If $P = 20$ and $l = 7$, what is w?

As outcomes, Year 9 pupils should, for example:

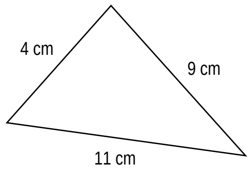
Explain the meaning of and substitute numbers into formulae such as:

- A formula to convert C degrees Celsius to F degrees Fahrenheit:
  $F = \frac{9C}{5} + 32$ or $F = \frac{9}{5}(C + 40) - 40$
- The Greek mathematician Hero showed that the area A of a triangle with sides a, b and c is given by the formula

  $A = \sqrt{s(s - a)(s - b)(s - c)}$,

  where $s = \frac{1}{2}(a + b + c)$.

  Use Hero's formula to find the area of this triangle.

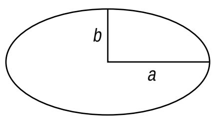


Find an unknown where it is not the subject of the formula and where an equation must be solved. For example:

- The surface area S of a cuboid of length l, width w and height h is
  $S = 2lw + 2lh + 2hw$

  What is h if $S = 410$, $l = 10$ and $w = 5$?

- The area A cm^2 enclosed by an ellipse is given by
  $A = \pi ab$



  Calculate to one decimal place the length a cm, if $b = 3.2$ and $A = 25$.

In simple cases, change the subject of a formula, using inverse operations. For example:

- Make I or R the subject of the formula
  $V = IR$
- Make l or w the subject of the formula
  $P = 2(l + w)$
- Make b or h the subject of the formulae
  $A = \frac{1}{2}bh$ $V = lbh$
- Make r the subject of the formulae
  $C = 2\pi r$ $A = \pi r^2$
  $V = \pi r^2 h$ $V = \frac{4}{3}\pi r^3$
- Make u, a or t the subject of the formula
  $v = u + at$
- Make C the subject of the formula
  $F = \frac{9C}{5} + 32$

- *Make l the subject of the formula*
  $T = 2\pi\sqrt{(l/g)}$
- *Make u or v the subject of the formula*
  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Pupils should be taught to:

Use formulae from mathematics and other subjects (continued)

As outcomes, Year 7 pupils should, for example:

Derive simple algebraic expressions and formulae.
Check for correctness by substituting particular values.

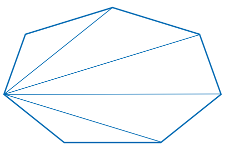
For example:

- You have p pencils.
  a. Rashida has twice as many pencils as you have. How many pencils does Rashida have?
  b. You give away 2 pencils. How many pencils do you have left?
  c. Rashida shares her pencils equally between herself and 4 other friends. How many pencils do they each get?

- Jo plants cherry trees, plum trees, apple trees and pear trees. n stands for the number of cherry trees Jo plants.
  a. Jo plants the same number of plum trees as cherry trees. How many plum trees does she plant?
  b. Jo plants twice as many apple trees as cherry trees. How many apple trees does she plant?
  c. Jo plants 7 more pear trees than cherry trees. How many pear trees does she plant?
  d. How many trees does Jo plant altogether? Write your answer as simply as possible.

Derive formulae such as:

- the number c of connecting lines joining m dots to n dots:
  $c = m \times n$
- the number s of 1-metre square concrete slabs that will surround a rectangular ornamental pond that is 1 metre wide and m metres long:
  $s = 2m + 6$
- the number D of (non-intersecting) diagonals from a single vertex in a polygon with n sides:
  $D = n - 3$



Link to finding the *n*th term of a sequence (pages 154–7).

Use a **spreadsheet** to construct simple formulae to model situations such as:

- the number of pints in a gallon;
- a currency conversion chart for use when going on a foreign holiday.

	A	B	C	D	E	F	G	H
1	£ (pounds sterling)	10	=B1+10	=C1+10	=D1+10	=E1+10	=F1+10	=G1+10
2	$ (US dollars)	=B1*1.6	=C1*1.6	=D1*1.6	=E1*1.6	=F1*1.6	=G1*1.6	=H1*1.6

	A	B	C	D	E	F	G	H
1	£ (pounds sterling)	10	20	30	40	50	60	70
2	$ (US dollars)	16	32	48	64	80	96	112

As outcomes, Year 8 pupils should, for example:

Derive algebraic expressions and formulae.
Check by substituting particular values.

For example:

- Mr Varma bought n apples and some oranges.
  a. He had 4 times as many oranges as apples. How many oranges did he have?
  b. He had 3 oranges left after making a pudding. How many oranges did he use?
  c. He used half the apples in a pie and his son ate one. How many apples were left?

- José has x euros and Juan has y euros.
  Write equations for each of these statements.
  a. José and Juan have a total of 2000 euros.
  b. Juan has four times as many euros as José.
  c. If Juan gave away 400 euros he would then have three times as many euros as José.
  d. If Juan gave 600 euros to José they would both have the same number of euros.
  e. Half of José's euros is the same as two fifths of Juan's.

Derive formulae such as:

- the number f of square faces that can be seen by examining a stack of n cubes:
  $f = 4n + 2$

- the sum S of the interior angles of a polygon with n sides:
  $S = (n - 2) \times 180°$

- the area A of a parallelogram with base b and height h:
  $A = b \times h$

- the number n half way between two numbers n_1 and n_2:
  $n = \dfrac{n_1 + n_2}{2}$

Link to finding the *n*th term of a sequence (pages 154–7).

Use a **spreadsheet** to construct simple formulae to model situations such as:

- petrol for a car that uses 1 litre for every 8 miles;
- sale prices at 5% discount.

	A	B	C
1	Original price (£)	5% discount (£)	Sale price (£)
2	50	=0.05*A2	=A2–B2
3	=A2+5	=0.05*A3	=A3–B3
4	=A3+5	=0.05*A4	=A4–B4
5	=A4+5	=0.05*A5	=A5–B5
6	=A5+5	=0.05*A6	=A6–B6

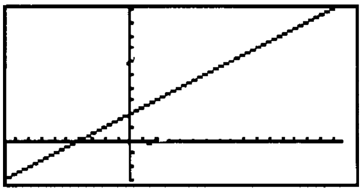
	A	B	C
1	Original price (£)	5% discount (£)	Sale price (£)
2	50	2.5	47.5
3	55	2.75	52.25
4	60	3	57
5	65	3.25	61.75
6	70	3.5	66.5

As outcomes, Year 9 pupils should, for example:

Derive more complex algebraic expressions and formulae. Check by substituting particular values.

For example:

- To cook a chicken allow 20 minutes per ½ kg and another 20 minutes.
  A chicken weighs x kg.
  Write an expression to show the number of minutes m to cook a chicken.



Link to conversion graphs (pages 172–3), and graphs of the form $y = mx + c$ (pages 164–9).

Derive formulae such as:

- Euler's formula for a plane network, where N = number of nodes, R = number of regions and A = number of arcs:
  $N + R = A + 2$

- the area A of a trapezium with parallel sides a and b, and height h:
  $A = ½(a + b) \times h$

- the area A of an annulus with outer radius r_1 and inner radius r_2:
  $A = \pi(r_1^2 - r_2^2)$

- the perimeter p of a semicircle with radius r:
  $p = r(\pi + 2)$

Link to finding the *n*th term of a sequence (pages 154–7).

Use a **spreadsheet** to construct formulae to model situations such as:

- the height of bounce when a rubber ball is dropped from varying heights;
- a mobile phone tariff based on a monthly rental charge plus the cost per minute of calls.

	A	B	C	D
1	Rent per month (£)	Cost of calls per minute (£)	Number of minutes	Tost cost (£)
2				=A2+(B2*C2)

	A	B	C	D
1	Rent per month (£)	Cost of calls per minute (£)	Number of minutes	Tost cost (£)
2	£15	£0.1	30	£18