Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals

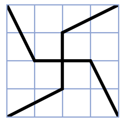
As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
numerator, denominator, mixed number, proper fraction, improper fraction… decimal fraction, percentage… equivalent, cancel, simplify, convert… lowest terms, simplest form…

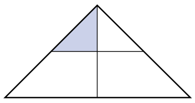
Understand a fraction as part of a whole.

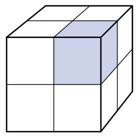
Use fraction notation to describe a proportion of a shape.
For example:

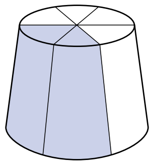
- Find different ways of dividing a 4 by 4 grid of squares into quarters using straight lines.



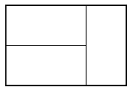
- Estimate the fraction of each shape that is shaded.







- Shade one half of this shape.



- Watch a **computer simulation** of a square being sectioned into fractional parts.
  Shade a fraction of a 6 by 6 grid of squares, e.g. one third.
  Convince a partner that exactly one third is shaded.

Relate fractions to division. Know that 4 ÷ 8 is another way of writing $\frac{4}{8}$, which is the same as $\frac{1}{2}$.

Express a smaller number as a fraction of a larger one.
For example:

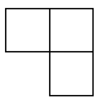
- What fraction of:
  - 1 metre is 35 centimetres?
  - 1 kilogram is 24 grams?
  - 1 hour is 33 minutes?
  - 1 yard is 1 foot?

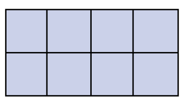
- What fraction of a turn does the minute hand turn through between:
  - 7:15 p.m. and 7:35 p.m.?
  - 3:05 p.m. and 6:50 p.m.?

- What fraction of a turn takes you from facing north to facing south-west?

- What fraction of a turn is 90°, 36°, 120°, 450°?

- What fraction of the big shape is the small one? ($\frac{3}{8}$)





Know the meaning of *numerator* and *denominator*.

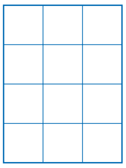
As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
terminating decimal, recurring decimal…
unit fraction…

Use fraction notation to describe a proportion of a shape. For example:

- Draw a 3 by 4 rectangle.

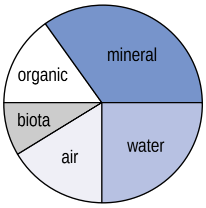
  Divide it into four parts that are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{12}$ of the whole rectangle. Parts must not overlap.

  Now draw a 4 by 5 rectangle. Divide it into parts. Each part must be a unit fraction of the whole rectangle, i.e. with numerator 1.

  Try a 5 by 6 rectangle. And a 3 by 7 rectangle?

- The pie chart shows the proportions of components in soil.

  Estimate the fraction of the soil that is:
  a. water; b. air.

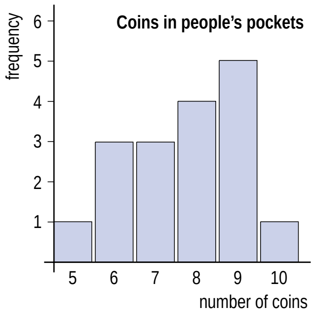
Relate fractions to division. Know that 43 ÷ 7 is another way of writing $\frac{43}{7}$, which is the same as $6\frac{1}{7}$.

Express a number as a fraction (in its lowest terms) of another. For example:

- What fraction of 180 is 120?

- What fraction of:
  1 foot is 3 inches?
  1 year is February?

- The bar chart shows the numbers of coins in people's pockets.
  What fraction of the total number of people had 7 coins in their pockets?

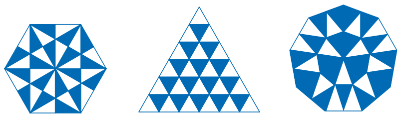
  

Link to enlargement and scale factor (pages 212–15).

As outcomes, Year 9 pupils should, for example:

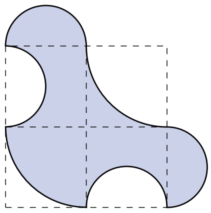
Use fraction notation to describe a proportion of a shape. For example:

- Estimate the fraction of each shape that is shaded.

- The curves of this shape are semicircles or quarter circles.

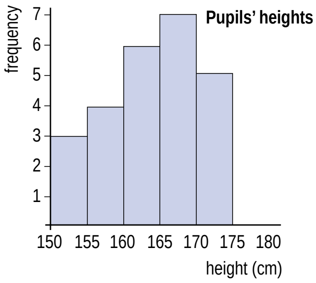
  Express the shaded shape as a fraction of the large dashed square.

Express a number as a fraction (in its lowest terms) of another. For example:

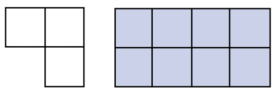
- What fraction of 120 is 180? ($\frac{3}{2}$ or $1\frac{1}{2}$)

- This frequency diagram shows the heights of a class of girls, classified in intervals $150 \le h < 155$, etc.

  What fraction of the girls are between 150 cm and 160 cm tall?

- What fraction of the small shape is the large one? ($\frac{8}{3}$ or $2\frac{2}{3}$)

Link to enlargement and scale factor (pages 212–15).

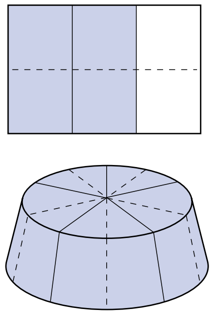
Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals (continued)

As outcomes, Year 7 pupils should, for example:

Simplify fractions by cancellation and recognise equivalent fractions.

Understand how equivalent fractions can be shown in diagrammatic form, with shapes sectioned into equal parts.



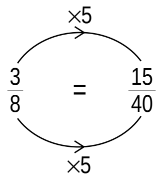
$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{5}{5} = 1$$

$$\frac{10}{10} = 1$$

Find equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

For example, recognise that because
$3 \times 5 = 15$
$8 \times 5 = 40$
it follows that $\frac{3}{8}$ is equivalent to $\frac{15}{40}$.



Know that if the numerator and the denominator have no common factors, the fraction is expressed in its lowest terms.

Answer questions such as:

- Cancel these fractions to their simplest form by looking for highest common factors:
  $\frac{9}{15}$ $\frac{12}{18}$ $\frac{42}{56}$
- Find two other fractions equivalent to $\frac{4}{5}$.
- Show that $\frac{12}{18}$ is equivalent to $\frac{6}{9}$ or $\frac{4}{6}$ or $\frac{2}{3}$.
- Find the unknown numerator or denominator in:
  $\frac{1}{4} = \frac{\square}{48}$ $\frac{7}{12} = \frac{35}{\square}$ $\frac{36}{24} = \frac{\square}{16}$

Link to finding the highest common factor (pages 54–5).

Continue to convert improper fractions to mixed numbers and vice versa: for example, change $\frac{34}{8}$ to $4\frac{1}{4}$, and $5\frac{7}{12}$ to $\frac{67}{12}$.

Answer questions such as:
- Convert $\frac{36}{5}$ to a mixed number.
- Which fraction is greater, $4\frac{1}{7}$ or $\frac{29}{7}$?
- How many fifths are there in $7\frac{1}{5}$?
- The fraction $\frac{7}{14}$ has three digits, 7, 1 and 4. It is equal to $\frac{1}{2}$. Find all the three-digit fractions that are equal to $\frac{1}{2}$. Explain how you know you have found them all.
- Find all the three-digit fractions that are equal to $\frac{1}{3}$. And $\frac{1}{4}$…
- There is only one three-digit fraction that is equal to $1\frac{1}{2}$. What is it?
- Find all the three-digit fractions that are equal to $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$…

See Y456 examples (pages 22–3).

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Understand the equivalence of algebraic fractions.

For example:

$$\frac{ab}{ac} \equiv \frac{b}{c}$$

$$\frac{ab^2}{abc} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{b}} \times b}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{b}} \times c} = \frac{b}{c}$$

$$\frac{3ab}{6bc} = \frac{a}{2c}$$

Simplify algebraic fractions by finding common factors. For example:

- *Simplify* $\frac{3a + 2ab}{4a^2}$

Recognise when cancelling is inappropriate. For example, recognise that:

- $\frac{a + b}{b}$ *is not equivalent to* $a + 1$;
- $\frac{a + b}{b}$ *is not equivalent to* a;
- $\frac{ab - 1}{b}$ *is not equivalent to* $a - 1$.

Link to adding algebraic fractions (pages 118–19).

Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals (continued)

As outcomes, Year 7 pupils should, for example:

Convert terminating decimals to fractions.
Recognise that each terminating decimal is a fraction:
for example, $0.27 = \frac{27}{100}$.

Convert decimals (up to two decimal places) to fractions.
For example:
- Convert 0.4 to $\frac{4}{10}$ and then cancel to $\frac{2}{5}$.
- Convert 0.32 to $\frac{32}{100}$ and then cancel to $\frac{8}{25}$.
- Convert 3.25 to $3\frac{25}{100} = 3\frac{1}{4}$.

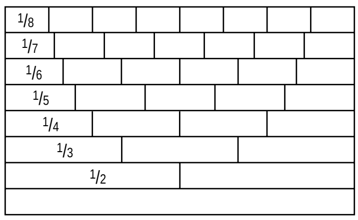
Link to place value (pages 36–9).

Convert fractions to decimals.
Convert a fraction to a decimal by using a known equivalent fraction. For example:
- $\frac{2}{8} = \frac{1}{4} = 0.25$
- $\frac{3}{5} = \frac{6}{10} = 0.6$
- $\frac{3}{20} = \frac{15}{100} = 0.15$

Convert a fraction to a decimal by using a known equivalent decimal. For example:
- Because $\frac{1}{5} = 0.2$
  $\frac{3}{5} = 0.2 \times 3 = 0.6$

See Y456 examples (pages 30–1).

Compare two or more simple fractions.
Deduce from a model or diagram that $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \dots$
and that, for example, $\frac{2}{3} < \frac{3}{4}$.



Answer questions such as:
- Insert a > or < symbol between each pair of fractions:
  $\frac{1}{2}$ ☐ $\frac{7}{10}$ $\frac{3}{8}$ ☐ $\frac{1}{2}$ $\frac{1}{2}$ ☐ $\frac{2}{3}$ $\frac{7}{15}$ ☐ $\frac{1}{2}$
- Write these fractions in order, smallest first:
  $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{5}{6}$;
  $2\frac{1}{10}$, $1\frac{3}{10}$, $2\frac{1}{2}$, $1\frac{1}{5}$, $1\frac{3}{4}$.
- Which of $\frac{5}{6}$ or $\frac{6}{5}$ is nearer to 1?
  Explain your reasoning.

See Y456 examples (pages 22–3).

As outcomes, Year 8 pupils should, for example:

Convert decimals to fractions.
Continue to recognise that each terminating decimal is a fraction. For example, $0.237 = \frac{237}{1000}$.

Recognise that a recurring decimal is a fraction.

Convert decimals (up to three decimal places) to fractions. For example:
- Convert 0.625 to $\frac{625}{1000}$ and then cancel to $\frac{5}{8}$.

Link to percentages (pages 70–1).

Convert fractions to decimals.
Use division to convert a fraction to a decimal, without and with a calculator. For example:
- Use short division to work out that:
  $\frac{1}{5} = 0.2$ $\frac{3}{8} = 0.375$ $\frac{27}{8} = \ldots$ $\frac{3}{7} = \ldots$
- Use a **calculator** to work out that $\frac{7}{53} = \ldots$

Investigate fractions such as $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{1}{11}$, … converted to decimals. For example:
- Predict what answers you will get when you use a **calculator** to divide:
  3 by 3, 4 by 3, 5 by 3, 6 by 3, and so on.

Order fractions.
Compare and order fractions by converting them to fractions with a common denominator or by converting them to decimals. For example, find the larger of $\frac{7}{8}$ and $\frac{4}{5}$:
- using common denominators:
  $\frac{7}{8}$ is $\frac{35}{40}$, $\frac{4}{5}$ is $\frac{32}{40}$, so $\frac{7}{8}$ is larger.
- using decimals:
  $\frac{7}{8}$ is 0.875, $\frac{4}{5}$ is 0.8, so $\frac{7}{8}$ is larger.

Use equivalent fractions or decimals to position fractions on a number line. For example:
- Mark fractions such as $\frac{2}{5}$, $\frac{6}{20}$, $\frac{3}{15}$, $\frac{18}{12}$ on a number line graduated in tenths, then on a line graduated in hundredths.

Answer questions such as:
- Which is greater, 0.23 or $\frac{3}{16}$?
- Which fraction is exactly half way between $\frac{3}{5}$ and $\frac{5}{7}$?

As outcomes, Year 9 pupils should, for example:

Know that a recurring decimal is an exact fraction.
Know and use simple conversions for recurring decimals to fractions. For example:
- $0.333\,333\ldots = \frac{1}{3}$ ($= \frac{3}{9}$)
- $0.666\,666\ldots = \frac{2}{3}$
- $0.111\,111\ldots = \frac{1}{9}$
- $0.999\,999\ldots = \frac{9}{9} = 1$

Convert recurring decimals to fractions in simple cases, using an algebraic method. *For example:*

- $z = 0.333\,333\ldots$ (1)
  $10z = 3.333\,333\ldots$ (2)
  Subtracting (1) from (2) gives:
  $9z = 3$
  $z = \frac{1}{3}$

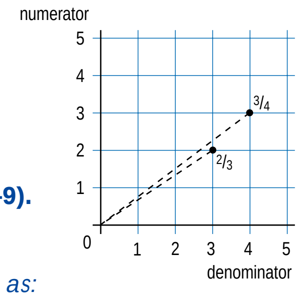
- *Comment on:*
  $z = 0.999\,999\ldots$
  $10z = 9.999\,999\ldots$
  $9z = 9$
  $z = 1$

Order fractions.
Answer questions such as:
- The numbers $\frac{1}{2}$, a, b, $\frac{3}{4}$ are in increasing order of size. The differences between successive numbers are all the same. What is the value of b?
- z is a decimal with one decimal place.
  Write a list of its possible values, if both these conditions are satisfied:
  $\frac{1}{3} < z < \frac{2}{3}$ $\frac{1}{6} < z < \frac{5}{6}$

Link to inequalities (pages 112–13).

Order fractions by graphing them.
Compare gradients.

Link to gradients (page 167–9).



Investigate sequences such as:

$$\frac{1}{2}, \quad \frac{2}{3}, \quad \frac{3}{4}, \quad \frac{4}{5}, \quad \ldots, \quad \frac{n}{n+1}$$

Investigate what happens as the sequence continues and n tends towards infinity.
Convert the fractions to decimals or draw a graph of the decimal against the term number.

Pupils should be taught to:

Calculate fractions of quantities; add, subtract, multiply and divide fractions

As outcomes, Year 7 pupils should, for example:

Add and subtract simple fractions.

Know addition facts for simple fractions, such as:

- $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
- $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$
- $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

and derive other totals from these results, such as:

- $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ (knowing that $\frac{1}{4} = \frac{2}{8}$)
- $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$ (knowing that $\frac{4}{8} = \frac{1}{2}$)

Add and subtract simple fractions with the same denominator. For example:

- $\frac{3}{8} + \frac{5}{8}$ $\frac{3}{5} + \frac{4}{5} + \frac{1}{5}$ $\frac{7}{10} + \frac{3}{10} + \frac{5}{10} + \frac{8}{10}$
- $\frac{6}{7} - \frac{4}{7}$ $\frac{9}{10} + \frac{4}{10} - \frac{3}{10}$

Begin to add and subtract simple fractions by writing them with a common denominator. For example:

- $\frac{5}{6} + \frac{1}{8} - \frac{7}{12} = \frac{20}{24} + \frac{3}{24} - \frac{14}{24} = \frac{20 + 3 - 14}{24} = \frac{9}{24}$

Calculate fractions of numbers, quantities or measurements.

Know that, for example:

- $\frac{1}{5}$ of 35 has the same value as 35 ÷ 5 = 7;
- $\frac{2}{3}$ of 15 has the same value as 15 ÷ 3 × 2 = 10;
- 0.5 of 18 has the same value as $\frac{1}{2}$ of 18 = 9.

Use mental methods to answer short questions with whole-number answers, such as:

- Find: one fifth of 40; two thirds of 150 g.
- Find: $\frac{1}{3}$ of 24; $\frac{3}{8}$ of 160; $\frac{9}{10}$ of 1 metre.
- Find: 0.5 of 50; 0.75 of 56; 1.25 of 40.

Use informal written methods to answer questions such as:

- If I make one fifth of a turn, how many degrees do I turn?
- Calculate: $\frac{7}{10}$ of £420; $\frac{6}{5}$ of 35; $\frac{3}{7}$ of 210; $1\frac{1}{4}$ of 2.4.

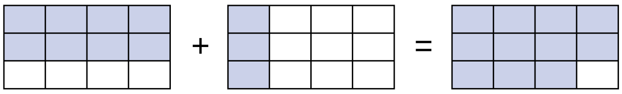
See Y456 examples (pages 24–5).

Link to multiplying fractions (pages 68–9).

As outcomes, Year 8 pupils should, for example:

Add and subtract fractions.

Use diagrams to illustrate adding and subtracting fractions, showing equivalence.



Know that fractions can only be added and subtracted if they have the same denominator. Use, for example, a single bar to avoid the problem of adding denominators:

$$\frac{3}{5} + \frac{5}{10} = \frac{6+5}{10} = \frac{11}{10} = 1\frac{1}{10}$$

Answer questions such as:

- Add/subtract these fractions:
  $\frac{1}{4} + \frac{5}{12}$ $\quad \frac{3}{5} + \frac{3}{4}$ $\quad \frac{5}{6} - \frac{3}{4}$
- Ancient Egyptian fractions were written with 1 as the numerator (unit fractions).
  Express these fractions as sums of unit fractions:
  $\frac{5}{8}$, $\frac{11}{12}$, $\frac{7}{10}$, $\frac{7}{12}$, $\frac{9}{20}$
- This fraction sum is made from four different digits, 1, 2, 4 and 8. The fraction sum is 1.

  $\frac{1}{2} + \frac{4}{8}$

  Find other fraction sums made from four different digits and with a fraction sum of 1.

Calculate fractions of numbers, quantities or measurements.

Develop written methods to answer short questions with fraction answers, such as:

- Find: three fifths of 17;
  two thirds of 140 g;
  $\frac{6}{25}$ of 34.

Link to multiplying and dividing fractions (pages 68–9).

As outcomes, Year 9 pupils should, for example:

Add and subtract fractions.

Add and subtract more complex fractions.
For example:

- $\frac{11}{18} + \frac{7}{24} = \frac{44+21}{72} = \frac{65}{72}$
- A photograph is $6\frac{1}{4}$ inches tall and $8\frac{5}{8}$ inches wide. Calculate its perimeter.
- *Investigate $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ and similar series.*
- *Begin to add and subtract algebraic fractions (pages 118–19), linking to number examples.*

Link to finding lowest common multiples (pages 54–5).

Calculate fractions of numbers, quantities or measurements.

Understand the multiplicative nature of fractions as operators. For example:

- Which is the greater: $\frac{3}{4}$ of 24 or $\frac{2}{3}$ of 21?
  34% of 75 or $\frac{5}{7}$ of 85?
  5 out of 16 or 8 out of 25?
- In a survey of 24 pupils, $\frac{1}{3}$ liked football best, $\frac{1}{4}$ liked basketball, $\frac{3}{8}$ liked athletics. The rest liked swimming. How many liked swimming?
- Brian used $\frac{1}{3}$ of a 750 g bag of flour to make scones. Claire used $\frac{2}{5}$ of the flour that remained to make a cake.
  How many grams of flour were left in the bag?
- In a bag of 20 coloured beads, $\frac{2}{5}$ are red, $\frac{1}{4}$ are blue, $\frac{1}{10}$ are yellow and 3 are green. The rest are black. What fraction are black?

Link to mutually exclusive events in probability (pages 278–81).

Pupils should be taught to:

Calculate fractions of quantities; add, subtract, multiply and divide fractions (continued)

As outcomes, Year 7 pupils should, for example:

Multiply a fraction by an integer or an integer by a fraction.

Know that $\frac{1}{4}$ of 12, $\frac{1}{4} \times 12$, $12 \times \frac{1}{4}$ and $12 \div 4$ are all equivalent.

Multiply a simple fraction by an integer. For example:

$\frac{1}{5} \times 3 = \frac{3}{5}$ $\frac{2}{5} \times 4 = \frac{8}{5}$

Simplify the product of a simple fraction and an integer. For example:

$\frac{1}{5} \times 15 = 3$

$\frac{2}{5} \times 15 = 2 \times \frac{1}{5} \times 15 = 2 \times 3 = 6$

$12 \times \frac{5}{6} = \frac{5}{6} \times 12 = 5 \times \frac{1}{6} \times 12 = 5 \times 2 = 10$

Answer questions such as:

- Find: $\frac{1}{9} \times 63$ $\frac{7}{9} \times 90$ $1\frac{1}{4} \times 10$
- Find: 0.25×24 0.2×50 3.3×40

As outcomes, Year 8 pupils should, for example:

Multiply an integer by a fraction.

Know that $\frac{2}{3}$ of 12, $\frac{2}{3} \times 12$ and $12 \times \frac{2}{3}$ are all equivalent.

Connect ordinary multiplication tables with patterns in fraction multiplication tables:

$\frac{1}{5} \times 1 = \frac{1}{5}$	$\frac{2}{5} \times 1 = \frac{2}{5}$	$\frac{3}{5} \times 1 = \frac{3}{5}$
$\frac{1}{5} \times 2 = \frac{2}{5}$	$\frac{2}{5} \times 2 = \frac{4}{5}$	$\frac{3}{5} \times 2 = \frac{6}{5}$
$\frac{1}{5} \times 3 = \frac{3}{5}$	$\frac{2}{5} \times 3 = \frac{6}{5}$	$\frac{3}{5} \times 3 = \frac{9}{5}$
$\frac{1}{5} \times 4 = \frac{4}{5}$	$\frac{2}{5} \times 4 = \frac{8}{5}$	$\frac{3}{5} \times 4 = \frac{12}{5}$

Think of multiplication by $\frac{1}{8}$ as division by 8, so $6 \times \frac{1}{8} = 6 \div 8$, and $6 \times \frac{3}{8} = 6 \times 3 \times \frac{1}{8} = 18 \div 8$.

Use cancellation to simplify the product of a fraction and an integer. For example:

$$\frac{7}{24} \times \frac{15}{1} = \frac{7}{\cancel{24}_{8}} \times \frac{\cancel{15}^{5}}{1} = \frac{35}{8}$$

Answer questions such as:

- Find: $\frac{3}{12} \times 30$ $\quad \frac{5}{9} \times 24$ $\quad 2\frac{1}{8} \times 10$

Understand that when multiplying a positive number by a fraction less than one, the result will be a smaller number. For example:
$24 \times \frac{1}{4} = 6$

Divide an integer by a fraction.

Know that a statement such as $24 \div \frac{1}{4}$ can be interpreted as:

- How many quarters are there in 24?
  $24 = \square \times \frac{1}{4}$ or $24 = \frac{1}{4} \times \square$.

For example:

- Look at one whole circle (or rectangle, prism…). How many sevenths can you see? (Seven.)
- Look at 1. How many fifths can you see? (Five.)
- Look at 4. How many fifths can you see? (Twenty.)
- Look at 4. How many two fifths can you see? (Ten.)

Use patterns. For example:

$60 \times \frac{1}{6} = 10$	and	$10 \div \frac{1}{6} = 60$
$30 \times \frac{2}{6} = 10$	and	$10 \div \frac{2}{6} = 30$
$20 \times \frac{3}{6} = 10$	and	$10 \div \frac{3}{6} = 20$
$15 \times \frac{4}{6} = 10$	and	$10 \div \frac{4}{6} = 15$
$12 \times \frac{5}{6} = 10$	and	$10 \div \frac{5}{6} = 12$

Understand that when dividing a positive number by a fraction less than one, the result will be a larger number. For example:
$24 \div \frac{1}{4} = 96$

As outcomes, Year 9 pupils should, for example:

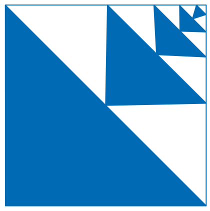
Multiply a fraction by a fraction.

Multiply fractions, using cancelling to simplify:

$$\frac{3}{4} \times \frac{2}{9} = \frac{\cancel{3}^{1} \times \cancel{2}^{1}}{\cancel{4}_{2} \times \cancel{9}_{3}} = \frac{1}{6}$$

For example:

- Calculate:
  a. $\frac{3}{5} \times \frac{20}{33} \times \frac{22}{14}$
  b. $\frac{22}{7} \times 14 \times 14$
  c. $4\frac{2}{3} \times 1\frac{3}{4}$
  d. $\frac{1}{2}(2 - \frac{1}{4})$
  e. $(2\frac{1}{2})^3$
- A photograph is $6\frac{1}{4}$ inches tall and $8\frac{5}{8}$ inches wide. Calculate its area.
- Imagine a square with sides of 1 metre. The area of the largest shaded triangle is $\frac{1}{2}$ m^2.

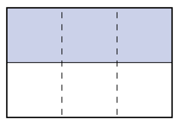
  a. Write the areas of the next two largest shaded triangles.

  b. *Use the diagram to help you find the sum of the infinite series:*
  $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \dots$
  Explain how you arrived at your solution.

Divide a fraction by a fraction.

Use the inverse rule to divide fractions, first converting mixed numbers to improper fractions. For example:

- Look at one half of a shape.

  How many sixths of the shape can you see? (Six.)
  So, how many sixths in one half? (Three.)
  So $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1} = \frac{6}{2} = 3$
- $\frac{2}{3} \div \frac{4}{7} = \frac{2}{3} \times \frac{7}{4} = \frac{14}{12}$ or $\frac{7}{6}$
- $2\frac{1}{3} \div \frac{4}{5} = \frac{7}{3} \times \frac{5}{4} = \frac{35}{12}$ or $2\frac{11}{12}$

Answer questions such as:

- Calculate: $(1 - \frac{1}{3})/(1 - \frac{5}{8})$
- The area of a circle is 154 cm^2.
  Taking π as $\frac{22}{7}$, find the radius of the circle.

Link to multiplying and dividing algebraic fractions (pages 118–19).

Pupils should be taught to:

Understand percentage as the number of parts per 100; recognise the equivalence of fractions, decimals and percentages; calculate percentages and use them to solve problems

As outcomes, Year 7 pupils should, for example:

Understand percentage as the number of parts in every 100, and express a percentage as an equivalent fraction or decimal. For example:

Convert percentages to fractions by writing them as the number of parts per 100, then cancelling. For example:

- 60% is equivalent to $\frac{60}{100} = \frac{3}{5}$;
- 150% is equivalent to $\frac{150}{100} = \frac{3}{2} = 1\frac{1}{2}$.

Convert percentages to decimals by writing them as the number of parts per 100, then using knowledge of place value to write the fraction as a decimal. For example:

- 135% is equivalent to $135 \div 100 = 1.35$.

Recognise the equivalence of fractions, decimals and percentages.

Know decimal and percentage equivalents of simple fractions.

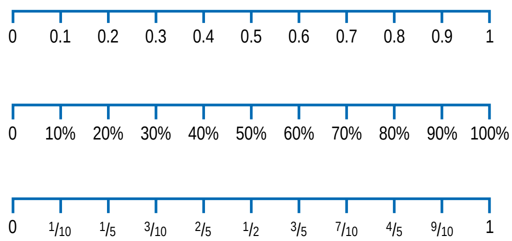
For example, know that $1 \equiv 100\%$. Use this to show that:

- $\frac{1}{10} = 0.1$ which is equivalent to 10%;
- $\frac{1}{100} = 0.01$ which is equivalent to 1%;
- $\frac{1}{8} = 0.125$ which is equivalent to $12\frac{1}{2}\%$;
- $1\frac{3}{4} = 1.75$ which is equivalent to 175%;
- $\frac{1}{3} = 0.333...$ which is equivalent to $33\frac{1}{3}\%$.

Express simple fractions and decimals as equivalent percentages by using equivalent fractions. For example:

- $\frac{3}{5} = \frac{60}{100}$ which is equivalent to 60%;
- $\frac{7}{20} = \frac{35}{100}$ which is equivalent to 35%;
- $2\frac{3}{4} = \frac{275}{100}$ which is equivalent to 275%;
- $0.48 = \frac{48}{100}$ which is equivalent to 48%;
- $0.3 = \frac{30}{100}$ which is equivalent to 30%.

Use number lines to demonstrate equivalence.



See Y456 examples (pages 32–3).

Link the equivalence of fractions, decimals and percentages to the probability scale (pages 278–9), and to the interpretation of data in pie charts and bar charts (pages 268–71).

As outcomes, Year 8 pupils should, for example:

Understand percentage as the operator 'so many hundredths of'.
For example, know that 15% means 15 parts per hundred, so 15% of Z means $^{15}/_{100} \times Z$.

Convert fraction and decimal operators to percentage operators by multiplying by 100.
For example:

- 0.45 $0.45 \times 100\% = 45\%$
- $^{7}/_{12}$ $(7 \div 12) \times 100\% = 58.3\%$ (to 1 d.p.)

Link the equivalence of fractions, decimals and percentages to the probability scale (pages 278–9), and to the interpretation of data in pie charts and bar charts (pages 268–71).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Understand percentage as the number of parts per 100; recognise the equivalence of fractions, decimals and percentages; calculate percentages and use them to solve problems (continued)

As outcomes, Year 7 pupils should, for example:

Calculate percentages of numbers, quantities and measurements.

Know that 10% is equivalent to $\frac{1}{10}$ = 0.1, and 5% is half of 10%.

Use **mental methods**. For example, find:
- 10% of £20 by dividing by 10;
- 10% of 37 g by dividing by 10;
- 5% of £5 by finding 10% and then halving;
- 100% of 5 litres by knowing that 100% represents the whole;
- 15% of 40 by finding 10% then 5% and adding the results together.

Use **informal written methods**. For example, find:

- 11% of £2800 by calculating 10% and 1% as jottings, and adding the results together;
- 70% of 130 g by calculating 10% and multiplying this by 7 as jottings; or by calculating 50% and 20% as jottings and adding the results.

Use a **calculator**, without the % key, to work out percentages of numbers and measures. For example:
- What is 24% of 34?
- Find 14.5% of 56 litres.

Know that there is more than one way to find a percentage using a calculator. For example, to find 12% of 45:

Convert a percentage calculation to an equivalent decimal calculation.

12% of 45 0.12×45

[.] [1] [2] [×] [4] [5] [=]

Convert a percentage calculation to an equivalent fraction calculation.

12% of 45 $\frac{12}{100} \times 45$

[1] [2] [÷] [1] [0] [0] [×] [4] [5] [=]

Recognise that this method is less efficient than the first.

Understand a calculator display when finding percentages in the context of money. For example:

- Interpret 15% of £48, displayed by most calculators as 7.2, as £7.20.

See Y456 examples (pages 32–3).

As outcomes, Year 8 pupils should, for example:

Calculate percentages of numbers, quantities and measurements.

Continue to use **mental methods**. For example, find:

- 65% of 40 by finding 50%, then 10% then 5% and adding the results together.
- 35% of 70 ml by finding 10%, trebling the result and then adding 5%;
- 125% of £240 by finding 25% then adding this to 240.

Use **written methods**. For example:

Use an equivalent fraction, as in:

- 13% of 48 $\frac{13}{100} \times 48 = \frac{624}{100} = 6.24$

Use an equivalent decimal, as in:

- 13% of 48 $0.13 \times 48 = 6.24$

Use a unitary method, as in:

- 13% of 48 1% of 48 = 0.48
  13% of 48 = $0.48 \times 13 = 6.24$

Use a **calculator**, without the % key, to work out percentages of numbers and measures.

Use an equivalent decimal calculation.

12% of 45 0.12×45

[.] [1] [2] [×] [4] [5] [=]

Use a unitary method; that is, find 1% first.

12% of 45 1% is 0.45, so 12% is 0.45×12

[.] [4] [5] [×] [1] [2] [=]

Recognise that these methods are equally efficient.

Extend understanding of the display on the calculator when using percentages of money.
For example:

- Interpret the answer to 33⅓% of £27, displayed by some calculators as 8.999999, as £9.

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Understand percentage as the number of parts per 100; recognise the equivalence of fractions, decimals and percentages; calculate percentages and use them to solve problems (continued)

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
change, total, value, amount, sale price, discount, decrease, increase, exchange rate, currency, convert...

Use the equivalence of fractions, decimals and percentages to compare two or more simple proportions and to solve simple problems.

Discuss percentages in everyday contexts. For example:
- Identify the percentage of wool, cotton, polyester... in clothes by examining labels.
- Work out what percentage of the pupils in the class are boys, girls, aged 11, have brown eyes...
- Discuss the use of percentages to promote the sales of goods, e.g. to indicate the extra amount in a packet.

Answer questions such as:

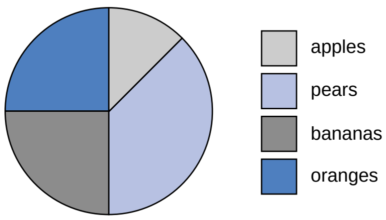
- Estimate the percentage of this line that is blue.

- 12% of a 125 g pot of yoghurt is whole fruit. How many grams are not whole fruit?

- 48% of the pupils at a school are girls. 25% of the girls and 50% of the boys travel to school by bus. What percentage of the whole school travels by bus?

Use proportions to interpret pie charts. For example:

- Some people were asked which fruit they liked best. This chart shows the results.



Estimate:
a. the percentage of the people that liked oranges best;
b. the proportion that liked apples best;
c. the percentage that did not choose pears.

Link to problems involving percentages (pages 2–3).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
profit, loss, interest, service charge, tax, VAT…
unitary method…

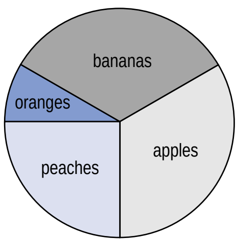
Use the equivalence of fractions, decimals and percentages to compare simple proportions and solve problems.

Apply understanding of simple percentages to other contexts, such as:

- the composition of alloys in science: for example, a 2p coin is 95% copper, 3.5% tin and 1.5% zinc;
- the age distribution of a population in geography;
- the elements of a balanced diet in nutrition;
- the composition of fabrics in design and technology: for example, a trouser fabric is 83% viscose, 10% cotton, 7% Lycra.

Answer questions such as:

- There is 20% orange juice in every litre of a fruit drink. How much orange juice is there in 2.5 litres of fruit drink? How much fruit drink can be made from 1 litre of orange juice?
- This chart shows the income that a market stall-holder got last week from selling different kinds of fruit.



  The stall-holder got £350 from selling bananas. Estimate how much she got from selling oranges.

- 6 out of every 300 paper clips produced by a machine are rejected. What is this as a percentage?
- Rena put £150 in her savings account. After one year, her interest was £12. John put £110 in his savings account. After one year, his interest was £12. Who had the better rate of interest, Rena or John? Explain your answer.

Link to problems involving percentages (pages 2–3).

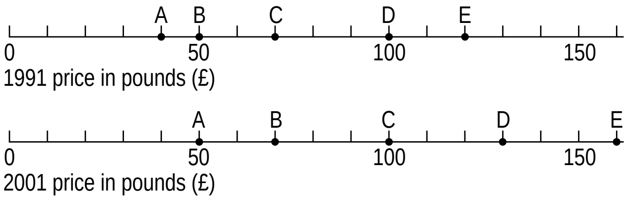
As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
cost price, selling price, compound interest…

Recognise when fractions or percentages are needed to compare proportions and solve problems.

Answer questions such as:

- A slogan on a tube of mints says that it is 23% bigger. It contains 20 mints. How many mints are there in the normal tube?
- Which is the better buy: a 400 g pack of biscuits at 52p, or a pack of biscuits with 400 g + 25% extra, at 57p?
- In a phone bill, VAT at 17.5% is added to the total cost of calls and line rental. What percentage of the total bill is VAT?
- In 1999, about 50% of the world's tropical rain forests had been destroyed. About 180 000 square kilomentres are now destroyed each year. This represents about 1.2% of the remainder. Estimate the original area of the tropical rain forests.
- The prices of five items A, B, C, D and E in 1991 and 2001 are shown on these scales.



  Which of the items showed the greatest percentage increase in price from 1991 to 2001?

Link to problems involving percentages (pages 2–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Understand percentage as the number of parts per 100; recognise the equivalence of fractions, decimals and percentages; calculate percentages and use them to solve problems (continued)	

As outcomes, Year 8 pupils should, for example:

Find the outcome of a given percentage increase or decrease.

Understand that:
- If something increases by 100%, it doubles.
- If something increases by 500%, it increases by five times itself, and is then six times its original size.
- A 100% decrease leaves zero.
- An increase of 15% will result in 115%, and 115% is equivalent to 1.15.
- A decrease of 15% will result in 85%, and 85% is equivalent to 0.85.
- An increase of 10% followed by a further increase of 10% is not equivalent to an increase of 20%.

For example:
- An increase of 15% on an original cost of £12 gives a new price of
  £12 × 1.15 = £13.80
  or
  15% of £12 = £1.80   £12 + £1.80 = £13.80

- A decrease of 15% on the original cost of £12 gives a new price of
  £12 × 0.85 = £10.20
  or
  15% of £12 = £1.80   £12 – £1.80 = £10.20

Investigate problems such as:

- I can buy a bicycle for one cash payment of £119, or pay a deposit of 20% and then six equal monthly payments of £17.
  How much extra will I pay in the second method?

- A price is increased by 10% in November to a new price. In the January sales the new price is reduced by 10%. Is the January sale price more, less or the same as the price was in October? Justify your answer.

- At the end of a dinner the waiter added VAT of 17.5% and then a 12.5% service charge. The customer argued that the service charge should have been calculated first.
  Who was correct?
  Give mathematical reasons for your answer.

Link to enlargement and scale (pages 212–17), and area and volume (pages 234–41).

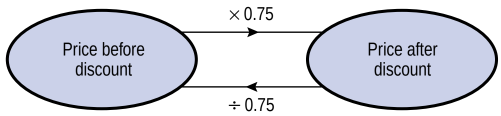
As outcomes, Year 9 pupils should, for example:

Use percentage changes to solve problems, choosing the correct numbers to take as 100%, or as a whole.

For example:
- There was a 25% discount in a sale.
  A boy paid £30 for a pair of jeans in the sale.
  What was the original price of the jeans?

  Using a unitary method
  £30 represents 75%.
  £30 ÷ 75 represents 1%.
  £30 ÷ 75 × 100 represents 100%.

  Using inverse operations
  Let p be the original price.
  $p \times 0.75 = 30$, so $p = 30 \div 0.75 = 40$



- An unstretched metal spring is 20 cm long.
  It is stretched to a length of 27 cm.
  Find the percentage change in its length.

  The increase is $\frac{7}{20} = \frac{35}{100}$ or 35%.

Solve problems such as:

- A jacket is on sale at £45, which is 85% of its original price. What was its original price?

- I bought a fridge freezer in a sale and saved £49. The label said that it was a ‘20% reduction’. What was the original price of the fridge freezer?

- A stereo system has been reduced from £320 to £272. What is the percentage reduction?

- The number of people going to a cinema increased from 52 000 in 1998 to 71 500 in 2001. Calculate the percentage increase in the number of people going to the cinema from 1998 to 2001.

- 12 500 people visited a museum in 2000. This was an increase of 25% on 1999. How many visitors were there in 1999?

- When heated, a metal bar increases in length from 1.25 m to 1.262 m. Calculate the percentage increase correct to one decimal place.

- *A woman deposits £75 in a bank with an annual compound interest rate of 6%. How much will she have at the end of 3 years? (The calculation $75 \times (1.06)^3$ gives the new amount.)*

Link to proportionality (pages 78–9), enlargement and scale (page 212–17), and area and volume (pages 234–41).

Pupils should be taught to:

Understand the relationship between ratio and proportion, and use ratio and proportion to solve simple problems

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
ratio, proportion…
and the notation 3 : 2.

Proportion compares part to whole, and is usually expressed as a fraction, decimal or percentage. For example:

- If there are 24 fish in a pond, and 6 are gold and 18 are black, there are 6 gold fish out of a total of 24 fish.
  The proportion of gold fish is 6 out of 24, or 1 in 4, or ¼, or 25%, or 0.25.

Solve problems such as:

- Tina and Fred each have some Smarties in a jar.
  The table shows how many Smarties they have, and how many of these Smarties are red.

	Number of Smarties	Number of red Smarties
Tina	440	40
Fred	540	45

  Who has the greater proportion of red Smarties, Tina or Fred?

Use direct proportion in simple contexts.
For example:

- Three bars of chocolate cost 90p.
  How much will six bars cost? And twelve bars?
- 1 litre of fruit drink contains 200 ml of orange juice.
  How much orange juice is there in 1.5 litres of fruit drink?
- £1 is worth 1.62 euros.
  How many euros will I get for £50?
- Here are the ingredients for fish pie for two people.

  Fish pie for two people

  250 g fish
  400 g potato
  25 g butter

  I want to make a fish pie for three people.
  How many grams of fish should I use?

See Y456 examples (pages 26–7).

Link to problems involving proportion (pages 4–5).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
direct proportion…

Solve simple problems involving direct proportion.
For example:

- 5 miles is approximately equal to 8 km.
  Roughly, how many km are equal to 20 miles?
  Roughly, how many miles are equal to 24 km?

  1 mile ≈ $\frac{8}{5}$ km

  20 miles ≈ $\frac{8}{5} \times 20$ km = 32 km

- 8 pizzas cost £16.
  What will 6 pizzas cost?

- 6 stuffed peppers cost £9.
  What will 9 stuffed peppers cost?

Link to problems involving proportion (pages 4–5).

Use a **spreadsheet** to explore direct proportion.
For example:

	A	B
1	No. of peppers	Cost (£)
2	1	=0.45*A2
3	2	=0.45*A3
4	3	=0.45*A4
5	4	=0.45*A5
6	5	=0.45*A6

	A	B
1	£	$
2	10	=1.62*A2
3	20	=1.62*A3
4	30	=1.62*A4
5	40	=1.62*A5
6	50	=1.62*A6

Link to conversion graphs (pages 172–3, 270–1), graphs of linear relationships (pages 164–5), and problems involving ratio and proportion (pages 4–5).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
proportionality, proportional to…
and the symbol ∝ (directly proportional to).

Identify when proportional reasoning is needed to solve a problem. For example:

- A recipe for fruit squash for six people is:

300 g	chopped oranges
1500 ml	lemonade
750 ml	orange juice

  Trina made fruit squash for ten people.
  How many millilitres of lemonade did she use?

  Jim used 2 litres of orange juice for the same recipe. How many people was this enough for?

Link to problems involving proportion (pages 4–5).

Use a **spreadsheet** to develop a table with a constant multiplier for linear relationships. Plot the corresponding graph using a **graph plotter** or **graphical calculator**.

Understand and use proportionality. *Use*
$y \propto x$ $y \propto x^2$ $y \propto 1/x$
to explore relationships between variables.

Use a ***spreadsheet*** *to test whether one set of numbers is directly proportional to another, e.g.*

	A	B	C	D	E	F	G	H
1	No. of litres	1	2	3	4	5	6	7
2	Price (p)	91	182	273	364	455	546	637
3	Price/litres	=B2/B1	=C2/C1	=D2/D1	=E2/E1	=F2/F1	=G2/G1	=H2/H1

Plot the corresponding graph using a ***graph plotter****.*

Compare with a non-linear relationship, such as
area of square = (side length)²

Use proportionality in other contexts. For example, from science know that pressure is proportional to force and weight is proportional to mass.

Appreciate that some 'real-life' relationships, particularly in science, may appear to be directly proportional but are not. For example, consider:

- *A plant grows 5 cm in 1 week.*
  How much will it grow in 1 year?
- *A man can run 1 mile in 4 minutes.*
  How far can he run in 1 hour?

Link to graphs of functions (pages 170–1).

Pupils should be taught to:

Understand the relationship between ratio and proportion, and use ratio and proportion to solve simple problems (continued)

As outcomes, Year 7 pupils should, for example:

Understand the idea of a ratio and use ratio notation.

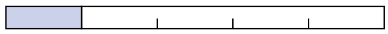
Ratio compares part to part. For example:
- If Lee and Ann divide £100 in the ratio 2 : 3, Lee gets 2 parts and Ann gets 3 parts. 1 part is £100 ÷ 5 = £20.
  So Lee gets £20 × 2 = £40 and Ann gets £20 × 3 = £60.

Know that the ratio 3 : 2 is not the same as the ratio 2 : 3.
- If Lee and Ann divide £100 in the ratio 3 : 2,
  Lee gets £60 and Ann gets £40.

Simplify a (two-part) ratio to an equivalent ratio by cancelling, e.g.
- Which of these ratios is equivalent to 3 : 12?
  A. 3 : 1 B. 9 : 36 C. 4 : 13 D. 1 : 3

Link to fraction notation (pages 60–3).

Understand the relationship between ratio and proportion, and relate them both to everyday situations. For example:

- In this stick 
  the ratio of blue to white is one part to four parts or 1 : 4.
  The proportion of blue in the whole stick is 1 out of 5, and ⅕ or 20% of the whole stick is blue.

Divide a quantity into two parts in a given ratio and solve simple problems using informal strategies. For example:

- A girl spent her savings of £40 on books and clothes in the ratio 1 : 3.
  How much did she spend on clothes?
- Coffee is made from two types of beans, from Java and Colombia, in the ratio 2 : 3.
  How much of each type of bean will be needed to make 500 grams of coffee?
- 28 pupils are going on a visit.
  They are in the ratio of 3 girls to 4 boys.
  How many boys are there?

See Y456 examples (pages 26–7).

Link to problems involving ratio (pages 4–5).

Use simple ratios when interpreting or sketching maps in geography or drawing to scale in design and technology.

As outcomes, Year 8 pupils should, for example:

Simplify a (three-part) ratio to an equivalent ratio by cancelling. For example:
- Write the ratio 12 : 9 : 3 in its simplest form.

Link to fraction notation (pages 60–3).

Simplify a ratio expressed in different units. For example:
- 2 m : 50 cm
- 450 g : 5 kg
- 500 mm : 75 cm : 2.5 m

Link to converting between measures (pages 228–9).

Consolidate understanding of the relationship between ratio and proportion. For example:

- In a game, Tom scored 6, Sunil scored 8, and Amy scored 10. The ratio of their scores was 6 : 8 : 10, or 3 : 4 : 5. Tom scored a proportion of $\frac{3}{12}$ or $\frac{1}{4}$ or 25% of the total score.

Divide a quantity into two or more parts in a given ratio. Solve simple problems using a unitary method.

- Potting compost is made from loam, peat and sand, in the ratio 7 : 3 : 2 respectively. A gardener used 1½ litres of peat to make compost. How much loam did she use? How much sand?
- The angles in a triangle are in the ration 6 : 5 : 7. Find the sizes of the three angles.
- Lottery winnings were divided in the ratio 2 : 5. Dermot got the smaller amount of £1000. How much in total were the lottery winnings?

2 parts	=	£1000
1 part	=	£500
5 parts	=	£2500
Total	=	£1000 + £2500 = £3500

Link to problems involving ratio (pages 4–5).

Use ratios when interpreting or sketching maps or drawing to scale in geography and other subjects.

- A map has a scale of 1 : 10 000. What distance does 5 cm on the map represent in real life?

Link to enlargement and scale (pages 212–17).

As outcomes, Year 9 pupils should, for example:

Simplify a ratio expressed in fractions or decimals. For example:
- Write 0.5 : 2 in whole-number form.

Compare ratios by changing them to the form *m* : 1 or 1 : *m*. For example:
- The ratios of Lycra to other materials in two stretch fabrics are 2 : 25 and 3 : 40. By changing each ratio to the form 1 : *m*, say which fabric has the greater proportion of Lycra.
- The ratios of shots taken to goals scored by two hockey teams are 17 : 4 and 13 : 3 respectively. By changing each ratio to the form *m* : 1, say which is the more accurate team.

Interpret and use ratio in a range of contexts. For example:

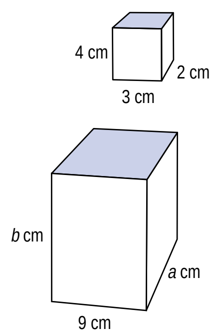
- Shortcrust pastry is made from flour and fat in the ratio 2 : 1. How much flour will make 450 g of pastry?
- An alloy is made from iron, copper, nickel and aluminium in the ratio 5 : 4 : 4 : 1. Find how much copper is needed to mix with 85 g of iron.
- 2 parts of blue paint mixed with 3 parts of yellow paint makes green. A boy has 50 ml of blue paint and 100 ml of yellow. What is the maximum amount of green he can make?
- On 1st June the height of a sunflower was 1 m. By 1st July, the height had increased by 40%. What was the ratio of the height of the sunflower on 1st June to its height on 1st July?

Link to problems involving ratio (pages 4–5).

Understand the implications of enlargement for area and volume. *For example:*

- *Corresponding lengths in these similar cuboids are in the ratio 1 : 3.*

  Find the values of a and b.
  Find the ratio of the areas of the shaded rectangles.
  Find the ratio of the volumes of the cuboids.



Link to enlargement and scale (pages 212–17).