Pupils should be taught to:

Use accurately the vocabulary, notation and labelling conventions for lines, angles and shapes; distinguish between conventions, facts, definitions and derived properties

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
line segment, line… parallel, perpendicular… plane…
horizontal, vertical, diagonal… adjacent, opposite…
point, intersect, intersection… vertex, vertices… side…
angle, degree (°)… acute, obtuse, reflex…
vertically opposite angles… base angles…

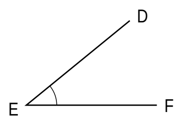
Use accurately the notation and labelling conventions for lines, angles and shapes.

Understand that a straight line can be considered to have infinite length and no measureable width, and that a line segment is of finite length, e.g. line segment AB has end-points A and B.

Know that:

- Two straight lines in a plane (a flat surface) can cross once or are parallel; if they cross, they are said to intersect, and the point at which they cross is an intersection.

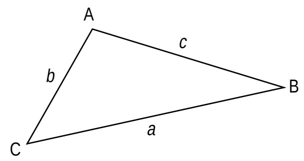
- When two line segments meet at a point, the angle formed is the measure of rotation of one of the line segments to the other. The angle can be described as ∠DEF or DÊF or ∠E.



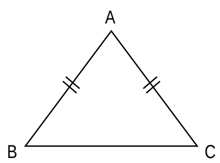
- A polygon is a 2-D or plane shape constructed from line segments enclosing a region. The line segments are called sides; the end points are called vertices. The polygon is named according to the number of its sides, vertices or angles: triangle, quadrilateral, pentagon…

Know the labelling convention for:

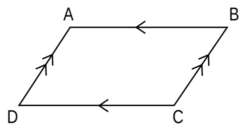
- triangles – capital letters for the vertices (going round in order, clockwise or anticlockwise) and corresponding lower-case letters for each opposite side, the triangle then being described as ∆ABC;



- equal sides and parallel sides in diagrams.



AB = AC



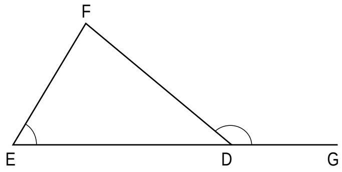
AB is parallel to DC, or AB//DC.
AD is parallel to BC, or AD//BC.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
corresponding angles, alternate angles…
supplementary, complementary…
interior angle, exterior angle… equidistant…
prove, proof…

Continue to use accurately the notation and labelling conventions for lines, angles and shapes.

Know that ∠DEF is an interior angle of ΔDEF and that ∠GDF is an exterior angle of ΔDEF.



Know that:
- A pair of complementary angles have a sum of 90°.
- A pair of supplementary angles have a sum of 180°.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
convention, definition, derived property…

Distinguish between conventions, definitions and derived properties.

A convention is an agreed way of illustrating, notating or describing a situation. Conventions are arbitrary – alternatives could have been chosen. Examples of geometrical conventions are:
- the ways in which letters are used to label the angles and sides of a polygon;
- the use of arrows to show parallel lines;
- the agreement that anticlockwise is taken as the positive direction of rotation.

A definition is a minimum set of conditions needed to specify a geometrical term, such as the name of a shape or a transformation. Examples are:
- A *polygon* is a closed shape with straight sides.
- A *square* is a quadrilateral with all sides and all angles equal.
- A *degree* is a unit for measuring angles, in which one complete rotation is divided into 360 degrees.
- A *reflection* in 2-D is a transformation in which points (P) are mapped to images (P’), such that PP’ is at right angles to a fixed line (called the mirror line, or line of reflection), and P and P’ are equidistant from the line.

A derived property is not essential to a definition, but consequent upon it. Examples are:
- The angles of a triangle add up to 180°.
- A square has diagonals that are equal in length and that bisect each other at right angles.
- The opposite sides of a parallelogram are equal in length.
- Points on a mirror line reflect on to themselves.

Distinguish between a practical demonstration and a proof. *For example, appreciate that the angle sum property of a triangle can be demonstrated practically by folding the corners of a triangular sheet of paper to a common point on the base and observing the result. A proof requires deductive argument, based on properties of angles and parallels, that is valid for all triangles.*

Pupils should be taught to:

Identify properties of angles and parallel and perpendicular lines, and use these properties to solve problems

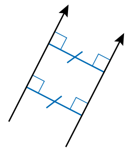
As outcomes, Year 7 pupils should, for example:

Identify parallel and perpendicular lines.

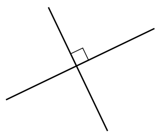
Recognise parallel and perpendicular lines in the environment, and in 2-D and 3-D shapes: for example, rail tracks, side edges of doors, ruled lines on a page, double yellow lines…

Use **dynamic geometry software**, acetate overlays or film to explore and explain relationships between parallel and intersecting lines, such as:

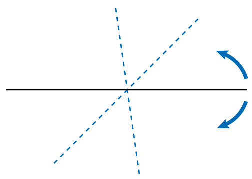
- parallel lines, which are always equidistant;



- perpendicular lines, which intersect at right angles;



- lines which intersect at different angles.
  For example, as one line rotates about the point of intersection, explain how the angles at the point of intersection are related.



Use ruler and set square to draw parallel and perpendicular lines.

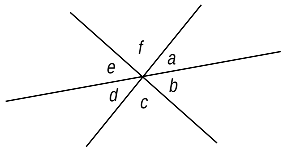
Link with constructions (page 220–3).

As outcomes, Year 8 pupils should, for example:

Identify alternate and corresponding angles.

Use **dynamic geometry software** or acetate overlays to explore and explain relationships between lines in the plane, such as:

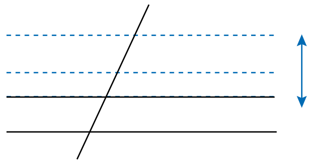
- three lines that intersect in one point;



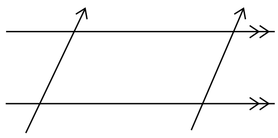
$a + b + c = 180°$

$a = d, \quad b = e, \quad c = f$

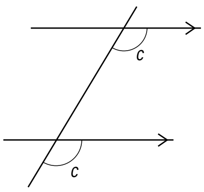
- given two intersecting lines and a third that moves but remains parallel to one of them, explain which angles remain equal;



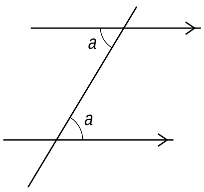
- two pairs of parallel lines, forming a parallelogram.



Understand and use the terms corresponding angles and alternate angles.



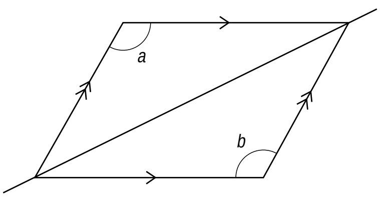
corresponding angles



alternate angles

Use alternate angles to prove that opposite angles of a parallelogram are equal:

$a = b$

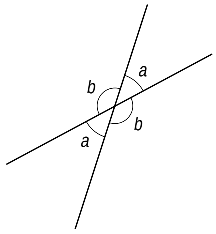


As outcomes, Year 9 pupils should, for example:

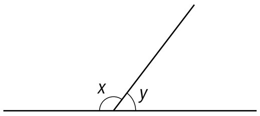
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Identify properties of angles and parallel and perpendicular lines, and use these properties to solve problems (continued)

Know the sum of angles at a point, on a straight line and in a triangle, and recognise vertically opposite angles and angles on a straight line.



vertically opposite angles



$x + y = 180°$

angles on a straight line

Link with rotation (pages 208–12).

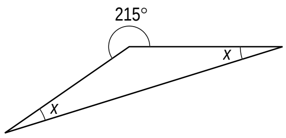
Recognise from practical work such as measuring and paper folding that the three angles of a triangle add up to 180°.

Given sufficient information, calculate:

- angles in a straight line and at a point;
- the third angle of a triangle;
- the base angles of an isosceles triangle.

For example:

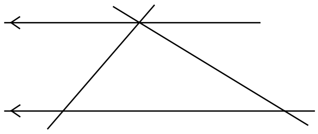
- Calculate the angles marked by letters.



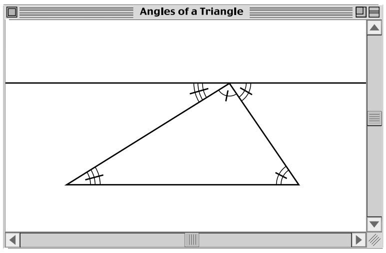
As outcomes, Year 8 pupils should, for example:

Understand a proof that the sum of the angles of a triangle is 180° and of a quadrilateral is 360°, and that the exterior angle of a triangle equals the sum of the two interior opposite angles.

Consider relationships between three lines meeting at a point and a fourth line parallel to one of them.



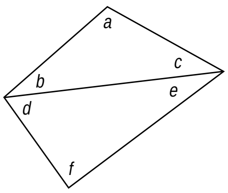
Use **dynamic geometry software** to construct a triangle with a line through one vertex parallel to the opposite side. Observe the angles as the triangle is changed by dragging any of its vertices.



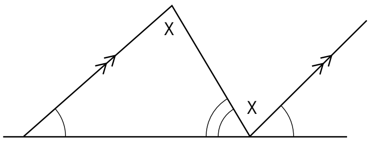
Use this construction, or a similar one, to explain using diagrams a proof that the sum of the three angles of a triangle is 180°.

Use the angle sum of a triangle to prove that the angle sum of a quadrilateral is 360°.

$(a + b + c) + (d + e + f)$
$= 180° + 180° = 360°$

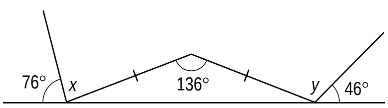


Explain a proof that the exterior angle of a triangle equals the sum of the two interior opposite angles, using this or another construction.



Given sufficient information, calculate:
- interior and exterior angles of triangles;
- interior angles of quadrilaterals.

For example:
- Calculate the angles marked by letters.

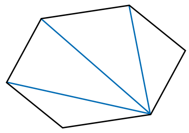


As outcomes, Year 9 pupils should, for example:

Explain how to find, calculate and use properties of the interior and exterior angles of regular and irregular polygons.

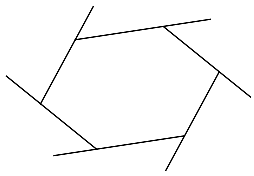
Explain how to find the interior angle sum and the exterior angle sum in (irregular) quadrilaterals, pentagons and hexagons. For example:

- A polygon with n sides can be split into $n - 2$ triangles, each with an angle sum of 180°.



So the interior angle sum is $(n - 2) \times 180°$, giving 360° for a quadrilateral, 540° for a pentagon and 720° for a hexagon.

At each vertex, the sum of the interior and exterior angles is 180°.

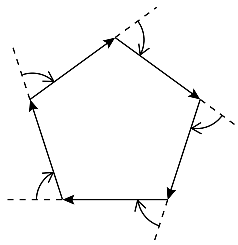


For n vertices, the sum of n interior and n exterior angles is $n \times 180°$.
But the sum of the interior angles is $(n - 2) \times 180°$, so the sum of the exterior angles is always $2 \times 180° = 360°$.

Find, calculate and use the interior and exterior angles of a regular polygon with n sides.
For example:

- The interior angle sum S for a polygon with n sides is $S = (n - 2) \times 180°$.
  In a regular polygon all the angles are equal, so each interior angle equals S divided by n.
  Since the interior and exterior angles are on a straight line, the exterior angle can be found by subtracting the interior angle from 180°.

- From experience of using **Logo**, explain how a complete traverse of the sides of a polygon involves a total turn of 360° and why this is equal to the sum of the exterior angles.



  Deduce interior angle properties from this result.

Recall that the interior angles of an equilateral triangle, a square and a regular hexagon are 60°, 90° and 120° respectively.

Pupils should be taught to:

Identify and use the geometric properties of triangles, quadrilaterals and other polygons to solve problems; explain and justify inferences and deductions using mathematical reasoning

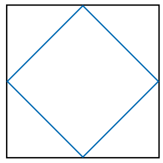
As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
polygon, regular, irregular, convex, concave…
circle, triangle, isosceles, equilateral, scalene, right-angled, quadrilateral, square, rectangle, parallelogram, rhombus, trapezium, kite, delta… and names of other polygons.

Visualise and sketch 2-D shapes in different orientations, or draw them using **dynamic geometry software**. Describe what happens and use the properties of shapes to explain why.

For example:

- Imagine a square with its diagonals drawn in. Remove one of the triangles. What shape is left? How do you know?
- Imagine a rectangle with both diagonals drawn. Remove a triangle. What sort of triangle is it? Why?
- Imagine joining adjacent mid-points of the sides of a square. What shape is formed by the new lines? Explain why.



- Imagine a square with one of its corners cut off. What different shapes could you have left?
- Imagine an isosceles triangle. Fold along the line of symmetry. What angles can you see in the folded shape? Explain why.
- Imagine a square sheet of paper. Fold it in half and then in half again, to get another smaller square. Which vertex of the smaller square is the centre of the original square? Imagine a small triangle cut off this corner. Then imagine the paper opened out. What shape will the hole be? Explain your reasoning.

  Imagine what other shapes you can get by folding a square of paper in different ways and cutting off different shapes.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
bisect, bisector, mid-point…
congruent… tessellate, tessellation…

Visualise and sketch 2-D shapes in different orientations, or draw them using **dynamic geometry software**. Describe what happens and use the properties of shapes to explain why.

For example:

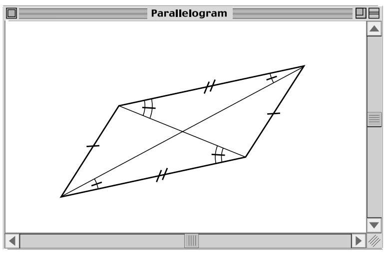
- Imagine a rectangular sheet of paper.
  Cut along the diagonal to make two triangles.
  Place the diagonals together in a different way.
  What shape is formed?

- Imagine two equilateral triangles, placed together, edge to edge.
  What shape is formed? Why?
  Add a third equilateral triangle… a fourth…
  What shapes are formed? Sketch some diagrams and explain what can be seen.

- Imagine two congruent isosceles triangles. Put sides of equal length together. Describe the resulting shape. Is it the only possibility?

- Imagine a quadrilateral with two lines of symmetry.
  What could it be?
  Suppose it also has rotation symmetry of order 2.
  What could it be now?

- Construct a parallelogram by drawing two line segments from a common end-point.
  Draw parallel lines to form the other two sides.
  Draw the two diagonals.



  Observe the sides, angles and diagonals as the parallelogram is changed by dragging its vertices.

- Describe tilings and other geometrical patterns in pictures and posters. Suggest reasons why objects in the environment (natural or constructed) take particular shapes.

- Explore tessellations using plastic or card polygon shapes and/or **computer tiling software**, and explain why certain shapes tessellate.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
similar, similarity…
hypotenuse, Pythagoras' theorem…

Visualise and sketch 2-D shapes or draw them using **dynamic geometry software** as they go through a sequence of changes. Describe what happens and use the properties of shapes to explain why.

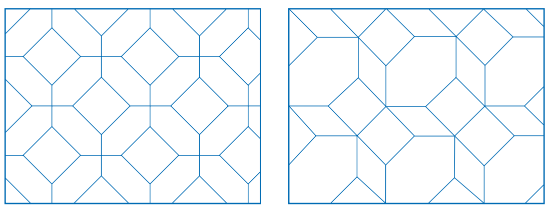
For example:

- Imagine starting with an equilateral triangle with one side horizontal – call it the base.
  Imagine this base is fixed.
  The opposite vertex of the triangle moves slowly in a straight line, perpendicular to the base.
  What happens to the triangle?
  Now imagine that the opposite vertex moves parallel to the base. What happens?
  Can you get a right-angled triangle, or an obtuse-angled triangle?

- Imagine a square sheet of paper.
  Imagine making a straight cut symmetrically across one corner. What shape is left?
  Imagine making a series of straight cuts, always parallel to the first cut. Describe what happens to the original square.

- Imagine two sheets of acetate, each marked with a set of parallel lines, spaced 2 cm apart.
  Imagine one sheet placed on top of the other, so that the two sets of lines are perpendicular.
  What shapes do you see?
  What happens to the pattern as the top sheet slowly rotates about a fixed point (the intersection of two lines)? What if the lines were 1 cm apart on one sheet and 2 cm apart on the other?

- Overlay tessellations in various ways, such as octagons and squares on octagons and squares.
  Describe the outcomes.



  Explore how regular polygons which do not tessellate (e.g. nonagons) can be used to cover the plane by leaving holes in a regular pattern.
  Describe the outcomes.

Pupils should be taught to:

Identify and use the geometric properties of triangles, quadrilaterals and other polygons to solve problems; explain and justify inferences and deductions using mathematical reasoning (continued)

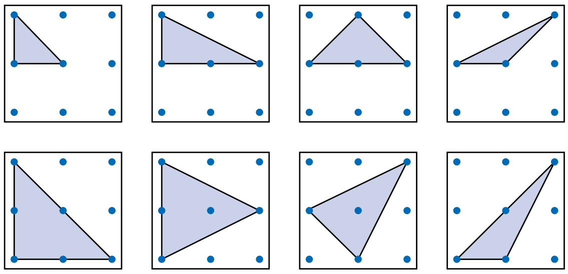
As outcomes, Year 7 pupils should, for example:

Triangles, quadrilaterals and other polygons

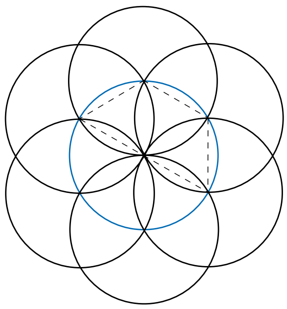
Review the properties of triangles and quadrilaterals
(see Y456 examples, pages 102–3).

For example:

- Using a 3 by 3 array on a pinboard, identify the eight distinct triangles that can be constructed (eliminating reflections, rotations or translations). Classify the triangles according to their side, angle and symmetry properties.



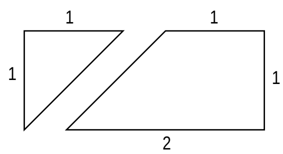
- In this pattern of seven circles, how many different triangles and quadrilaterals can you find by joining three or four points of intersection?



What are the names of the shapes and what can you find out about their angles?

Begin to identify and use angle, side and symmetry properties of triangles, quadrilaterals and other polygons. For example:

- Start with a 2 by 1 rectangle. Make these two shapes. What new shapes can you make with them? Name them and discuss their properties.



As outcomes, Year 8 pupils should, for example:

Know and use side, angle and symmetry properties of equilateral, isosceles and right-angled triangles.

For example:

- Discuss whether it is possible to draw or construct on a 3 by 3 pinboard:
  a. a triangle with a reflex angle;
  b. an isosceles trapezium;
  c. an equilateral triangle or a (non-square) rhombus.

  If not, explain why not.

Classify quadrilaterals by their geometric properties (equal and/or parallel sides, equal angles, right angles, diagonals bisected and/or at right angles, reflection and rotation symmetry…).

Know properties such as:

- An isosceles trapezium is a trapezium in which the two opposite non-parallel sides are the same length. It has one line of symmetry and both diagonals are the same length.
- A parallelogram has its opposite sides equal and parallel. Its diagonals bisect each other. It has rotation symmetry of order 2.
- A rhombus is a parallelogram with four equal sides. Its diagonals bisect each other at right angles. Both diagonals are lines of symmetry. It has rotation symmetry of order 2.
- A kite is a quadrilateral that has two pairs of adjacent sides of equal length, and no interior angle larger than 180°. It has one line of symmetry and its diagonals cross at right angles.
- An arrowhead or delta has two pairs of adjacent edges of equal length and one interior angle larger than 180°. It has one line of symmetry. Its diagonals cross at right angles outside the shape.

Provide a convincing argument to explain, for example, that a rhombus is a parallelogram but a parallelogram is not necessarily a rhombus.

Devise questions for a tree classification diagram to sort a given set of quadrilaterals.

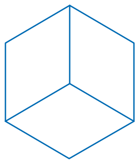
- Identify then classify the 16 distinct quadrilaterals that can be constructed on a 3 by 3 pinboard.

Link to standard constructions (pages 220–3).

As outcomes, Year 9 pupils should, for example:

Know and use angle and symmetry properties of polygons, and angle properties of parallel and intersecting lines, to solve problems and explain reasoning. For example:

- Deduce the angles of the rhombus in this arrangement of three identical tiles.



  What can you deduce about the shape formed by the outline?

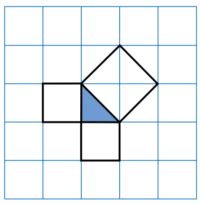
- Explain why:
  – Equilateral triangles, squares and regular hexagons will tessellate on their own but other regular polygons will not.
  – Squares and regular octagons will tessellate together.

Know and use properties of triangles, *including Pythagoras' theorem.*

Know that:

- In any triangle, the largest angle is opposite the longest side and the smallest angle is opposite the shortest side.
- In a right-angled triangle, the side opposite the right angle is the longest and is called the hypotenuse.

Understand, recall and use Pythagoras' theorem.
Explain special cases of Pythagoras' theorem in geometrical arrangements such as:



Pupils should be taught to:

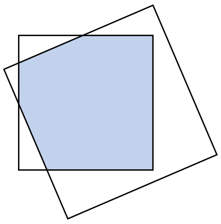
Identify and use the geometric properties of triangles, quadrilaterals and other polygons to solve problems; explain and justify inferences and deductions using mathematical reasoning (continued)

As outcomes, Year 7 pupils should, for example:

Use the properties of angles at a point and on a straight line, and the angle sum of a triangle, to solve simple problems. Explain reasoning.

For example:

- What different shapes can you make by overlapping two squares?



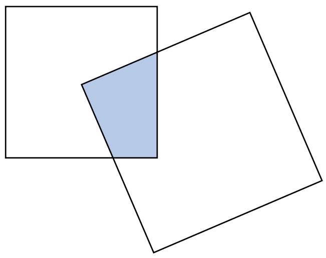
  Can any of these shapes be made?

rectangle	pentagon	decagon
rhombus	hexagon	kite
isosceles triangle	octagon	trapezium

  If a shape cannot be made, explain why.

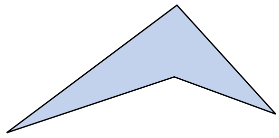
  What shapes can you make by overlapping three squares?

- Two squares overlap like this. The larger square has one of its vertices at the centre of the smaller square.



  Explain why the shaded area is one quarter of the area of the smaller square.

- Explain why a triangle can never have a reflex angle but a quadrilateral can.



- Provide a convincing argument to explain why it is always possible to make an isosceles triangle from two identical right-angled triangles.

- Use **Logo** to write instructions to draw a parallelogram.

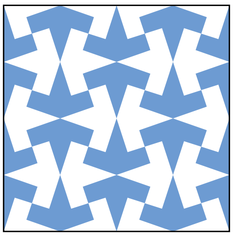
Link to problems involving shape and space (pages 14–17).

As outcomes, Year 8 pupils should, for example:

Use angle and side properties of triangles, and angle properties of parallel and intersecting lines, to solve problems. Explain reasoning. For example:

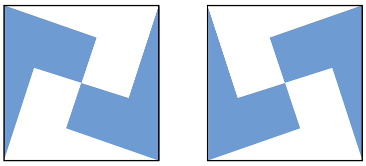
- Use alternate and corresponding angles to explain why any scalene triangle will tessellate.

- This tiling pattern can be found in the Alhambra Palace in Granada, Spain.



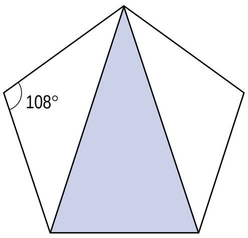
How would you describe the pattern over the telephone to someone who has never seen it?

The pattern can be made by using these two tiles.



Suggest how to construct them.
What other patterns can you make with these two tiles?
Reproduce the tiling pattern using **computer tiling software**.

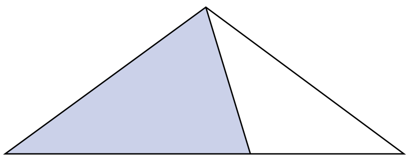
- The angle at the vertex of a regular pentagon is 108°.



Two diagonals are drawn to the same vertex to make three triangles.

Calculate the sizes of the angles in each triangle.

The middle triangle and one of the other triangles are placed together like this.



Explain why the triangles fit together to make a new triangle. What are its angles?

Link to problems involving shape and space (pages 14–17).

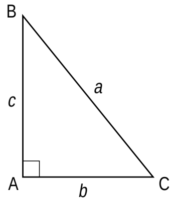
As outcomes, Year 9 pupils should, for example:

Understand and recall Pythagoras' theorem:

- *as a property of areas:*
  in a right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

- *as a property of lengths:*

  $a^2 = b^2 + c^2$



Appreciate that:

If $a^2 > b^2 + c^2$, then A is an obtuse angle.
If $a^2 < b^2 + c^2$, then A is an acute angle.

Know the Pythagorean triples (3, 4, 5) and (5, 12,13). Explore others using a ***spreadsheet*** *or* ***calculator****. Recognise that multiples of a Pythagorean triple are also Pythagorean triples and produce similar triangles.*

Link to problem solving – 'Hexagons' (pages 34–5), algebra (pages 120–1), trigonometry (pages 242–7), and coordinates (pages 218–19).

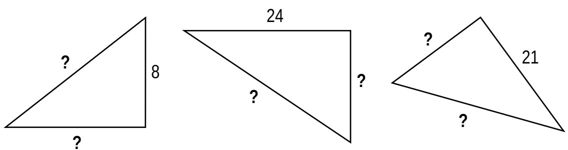
Use Pythagoras' theorem to solve simple problems in two dimensions. For example:

- *You walk due north for 5 miles, then due east for 3 miles. What is the shortest distance you are from your starting point?*

- *A 5 m ladder leans against a wall with its foot 1.5 m away from the wall. How far up the wall does the ladder reach?*

- *The sides of some triangles are:*
  a. *5, 12, 13*
  b. *6, 7, 8*
  c. *5, 8, 11*
  d. *16, 30, 34*
  e. *13, 15, 23*
  Without drawing the triangles, classify them according to whether they are acute-angled, right-angled or obtuse-angled.

- *Find whole-number lengths that will satisfy these right-angled triangles. There may be more than one answer.*



Link to problems involving shape and space (pages 14–17).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Understand congruence and similarity	

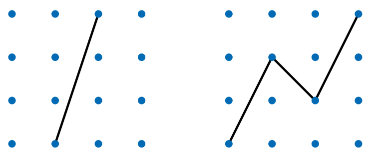
As outcomes, Year 8 pupils should, for example:

Congruence

Know that if two 2-D shapes are congruent, they have the same shape and size, and corresponding sides and angles are equal. For example:

- From a collection of different triangles or quadrilaterals, identify those that are congruent to each other by placing one on top of the other. Realise that corresponding sides and angles are equal.

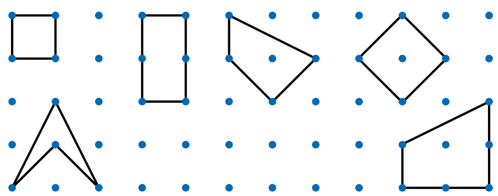
- Divide a 4 by 4 pinboard into two congruent halves. How many different ways of doing this can you find?



  Divide the pinboard into four congruent quarters.

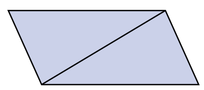
- Divide a 5 by 5 pinboard into two non-congruent halves.

- Using a 3 by 3 pinboard, make some different triangles or quadrilaterals. For each shape, investigate whether you can produce one or more identical shapes in different positions or orientations on the board. Describe the transformation(s) you use to do this.



  Extend to 3 by 4 and larger grids.

- Two congruent scalene triangles without right angles are joined with two equal edges fitted together.



  What shapes can result?
  What if the two triangles are right-angled, isosceles or equilateral?
  In each case, explain how you know what the resultant shapes are.

As outcomes, Year 9 pupils should, for example:

Congruence

Appreciate that when two shapes are congruent, one can be mapped on to the other by a translation, reflection or rotation, or some combination of these transformations.

See Year 8 for examples.

Link to transformations (pages 202–17).

Know from experience of constructing them that triangles satisfying SSS, SAS, ASA or RHS are unique, but that triangles satisfying SSA or AAA are not.

Link to constructions (pages 220–3).

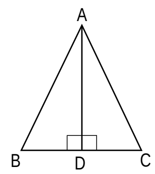
Appreciate that two triangles will be congruent if they satisfy the same conditions:

- *three sides are equal (SSS);*
- *two sides and the included angle are equal (SAS);*
- *two angles and a corresponding side are equal (ASA);*
- *a right angle, hypotenuse and side are equal (RHS).*

Use these conditions to deduce properties of triangles and quadrilaterals.

For example:

- *Draw triangle ABC, with AB = AC. Draw the perpendicular from A to BC to meet BC at point D.*



  Show that triangles ABD and ACD are congruent. Hence show that the two base angles of an isosceles triangle are equal.

- *Use congruence to prove that the diagonals of a rhombus bisect each other at right angles.*

- *By drawing a diagonal and using the alternate angle property, use congruence to prove that the opposite sides of a parallelogram are equal.*

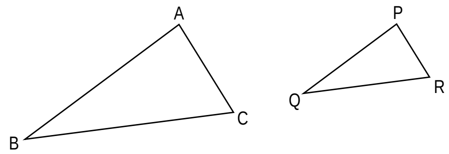
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Understand congruence and similarity (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Similarity

Know that the term 'similar' has a precise meaning in geometry. Objects, either plane or solid figures, are similar if they have the same shape. That is, corresponding angles contained in similar figures are equal, and corresponding sides are in the same ratio.



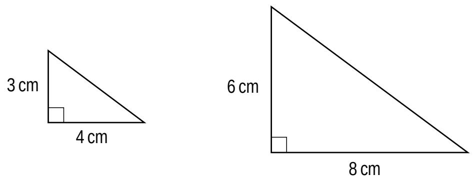
∠BAC = ∠QPR, ∠ACB = ∠PRQ, ∠ABC = ∠PQR and AB : PQ = BC : QR = CA : RP

Recognise and use characteristics of similar shapes.

- *If two shapes are similar, one can be considered to be an enlargement of the other.*
  Link to enlargement (pages 212–15).
- *Any two regular polygons with the same number of sides are mathematically similar (e.g. any two squares, any two equilateral triangles, any two regular hexagons).*
- *All circles are similar. Use this knowledge when considering metric properties of the circle, such as the relationship between circumference and diameter.*
  Link to circumference of a circle (pages 234–5).

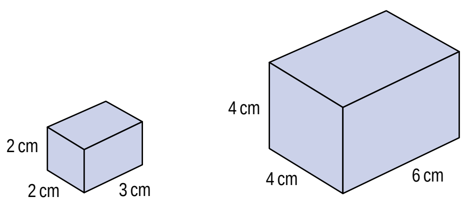
Solve problems involving similarity. For example:

- *These two triangles are similar.*



  a. *Find the perimeter and area of each triangle.*
  b. *What is the ratio of the two perimeters?*
  c. *What is the ratio of the two areas?*

- *Explain why these two cuboids are similar.*



  What is the ratio of their surface areas?
  What is the ratio of their volumes?

Link to ratio and proportion (pages 78–81).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Identify and use the properties of circles	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

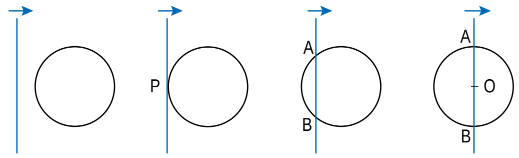
Circles

Know the parts of a circle, including:
centre, radius, diameter, circumference, chord, arc, segment, sector, tangent… and terms such as:
circumcircle, circumscribed, inscribed…

Know that:

- A circle is a set of points equidistant from its centre.
- The circumference is the distance round the circle.
- The radius is the distance from the centre to the circumference.
- An arc is part of the circumference.
- A sector is the region bounded an arc and two radii.

Use **dynamic geometry software** to show a line and a circle moving towards each other.

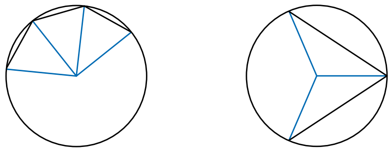


Know that when the line:

- touches the circle at a point P, it is called a tangent to the circle at that point;
- intersects the circle at two points A and B, the line segment AB is called a chord of the circle, which divides the area enclosed by the circle into two regions called segments;
- passes through the centre of the circle, the line segment AB becomes a diameter, which is twice the radius and divides the area enclosed by the circle into two semicircles.

Explain why inscribed regular polygons can be constructed by equal divisions of a circle.

- Appreciate that a chord of a circle, together with the radii through the end points, form an isosceles triangle. If further chords of the same length are drawn, the triangles are congruent and angles between successive chords are equal.

  Hence, if the succession of equal chords divides the circle without remainder, a regular polygon is inscribed in the circle.

- If chords of length equal to the radius are marked on the circumference of a circle, explain why the resultant shape is a regular hexagon.

  Use this to construct a hexagon of a given side.

Link to circumference and area of a circle (pages 234–7).

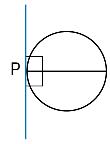
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Identify and use the properties of circles (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

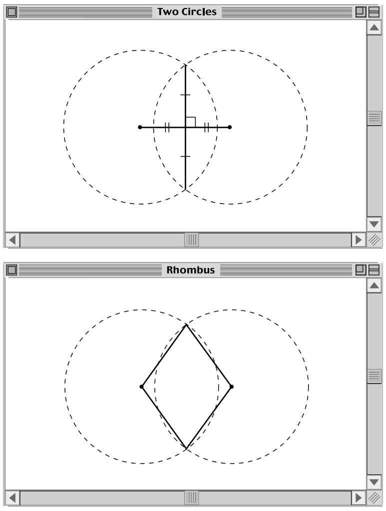
Circles (continued)

Recognise that a tangent is perpendicular to the radius at the point of contact P.

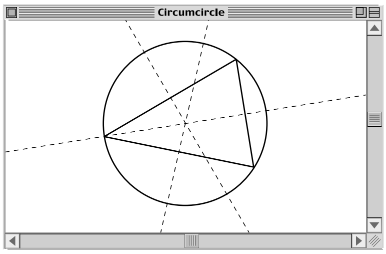


*Use **dynamic geometry software** to explore properties of lines and circles. For example:*

- *Make two circles, of equal radius, touch then intersect. What happens to the common chord and the line joining the centres, or to the rhombus formed by joining the radii to the points of intersection?*



- *Construct a triangle and the perpendicular bisectors of its three sides. Draw the circumcircle (the circle through the three vertices).*
  What happens when the vertices of the triangle move? Observe in particular the position of the centre of the circumcircle.



Link to standard constructions (pages 220–3).

Pupils should be taught to:

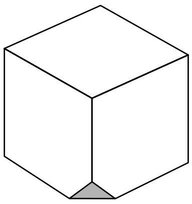
Use 2-D representations, including plans and elevations, to visualise 3-D shapes and deduce some of their properties

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
2-D, 3-D, cube, cuboid, pyramid, tetrahedron, prism, cylinder, sphere, hemisphere…
face, vertex, vertices, edge… net…

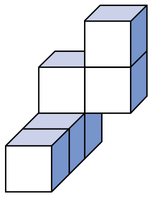
Use 2-D representations and oral descriptions to visualise 3-D shapes and deduce some of their properties.
For example:

- Imagine you have two identical cubes.
  Place them together, matching face to face.
  Name and describe the new solid.
  How many faces, edges, vertices…?



- Imagine cutting off a corner of a cube.
  Describe the new face created.
  Describe the new solid.
  How many faces, edges, vertices…?

- Sit back to back with a partner.
  Look at the picture of the model.
  Don't show it to your partner.
  Tell your partner how to build the model.

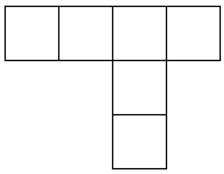


- Join in a 'guess the shape' activity.
  A solid made from centimetre cubes is placed in a bag.
  a. Take turns to describe one element of the shape by feeling it. Others try to make the same shape.
  b. Take turns to describe the shape while others try to guess what it is.
  c. Guess a hidden solid shape by asking questions about it to which only yes/no answers can be given.

- On a six-sided dice, the faces are numbered from 1 to 6, and opposite faces should add up to 7.

  Here is a net for a cube.
  Choose a face and write 5 on it.

  Now write numbers on the other faces so that when the cube is folded up, opposite faces add up to 7.



- Imagine you are visiting the pyramids in Egypt.
  You are standing on the ground, looking at one pyramid.
  What is the maximum number of faces you could see?
  What if you were flying overhead?

- Imagine you are looking at a large cardboard box in the shape of a cube.
  Can you stand so that you can see just one face?
  Sketch an outline of what you would see.
  Can you stand so that you can see just 2 faces, 3 faces, 4 faces? Sketch outlines of what you would see.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
view, plan, elevation… isometric…

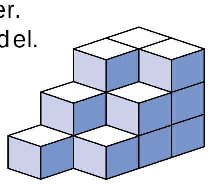
Know and use geometric properties of cuboids and shapes made from cuboids; begin to use plans and elevations. For example:

- Describe 3-D shapes which can be visualised from a wall poster or a photograph.

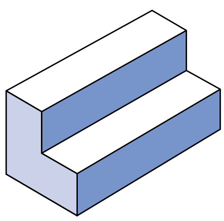
- Visualise and describe relationships between the edges of a cube, e.g. identify edges which:
  - meet at a point;
  - are parallel;
  - are perpendicular;
  - are neither parallel nor intersect each other.

- Imagine a cereal packet standing on a table.
  Paint the front and the back of the packet red.
  Paint the top and bottom red and the other two faces blue.
  Now study the packet carefully.
  How many edges has it?
  How many edges are where a red face meets a blue face?
  How many edges are where a red face meets another red face?
  How many edges are where a blue face meets another blue face?

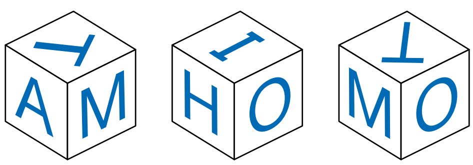
- Sit back to back with a partner.
  Look at the picture of the model.
  Don't show it to your partner.
  Tell your partner how to build the model.



- Sketch a net to make this model.
  Construct the shape.



- Here are three views of the same cube.
  Which letters are opposite each other?



As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
cross-section, projection…
plane…

Analyse 3-D shapes through 2-D projections and cross-sections, including plans and elevations.
For example:

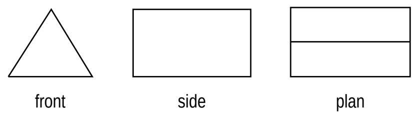
Visualise solids from an oral description. For example:

- In each case, identify the solid shape.
  a. The front and side elevations are both triangles and the plan is a square.
  b. The front and side elevations are both rectangles and the plan is a circle.
  c. The front elevation is a rectangle, the side elevation is a triangle and the plan is a rectangle.
  d. The front and side elevations and the plan are all circles.

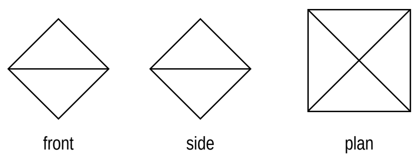
- The following are shadows of solids.
  Describe the possible solids for each shadow (there may be several solutions).

- In each case, identify the solid shape.
  Draw the net of the solid.

a.



b.



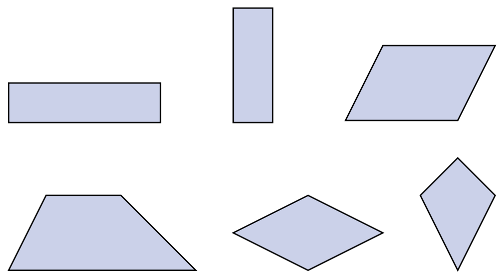
- Write the names of the polyhedra that could have an isosceles or equilateral triangle as a front elevation.

Pupils should be taught to:

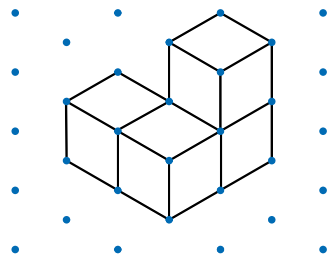
Use 2-D representations, including plans and elevations, to visualise 3-D shapes and deduce some of their properties (continued)

As outcomes, Year 7 pupils should, for example:

- A square piece of card is viewed from different angles against the light. Which of the following are possible views of the square card? Which are impossible?



- Find all possible solids that can be made from four cubes. Record the solids using isometric paper.



- Investigate the number of different ways that a 2 by 2 by 2 cube can be split into two pieces:
  a. of the same shape and size;
  b. of different shapes and sizes.

See Y456 examples (pages 104–5).

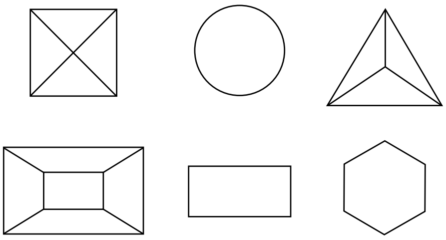
As outcomes, Year 8 pupils should, for example:

Orient an isometric grid and use conventions for constructing isometric drawings:
- Vertical edges are drawn as vertical lines.
- Horizontal edges are drawn at 30°.

Identify the position of hidden lines in an isometric drawing.

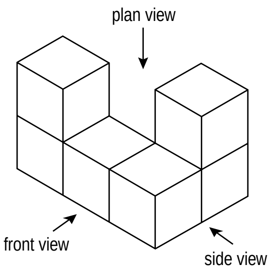
Begin to use plans and elevations. For example:

- The diagrams below are of solids when observed directly from above.

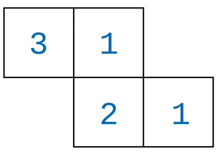


Describe what the solids could be and explain why.

- Draw the front elevation, side elevation and plan of this shape.

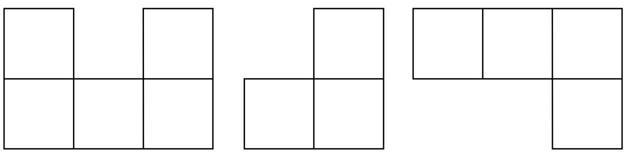


- This diagram represents a plan of a solid made from cubes, the number in each square indicating how many cubes are on that base.



Make an isometric drawing of the solid from a chosen viewpoint.

- Construct this solid, given the front elevation, side elevation and plan.



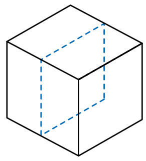
front elevation — side elevation — plan view

As outcomes, Year 9 pupils should, for example:

Visualise and describe sections obtained by slicing in different planes.
For example:

- Compare horizontal cross-sections of a square-based right pyramid at different heights.
  Repeat for vertical cross-sections at different points.

- This cube has been sliced to give a square cross-section.



Is it possible to slice a cube so that the cross-section is:
a. a rectangle?
b. a triangle?
c. a pentagon?
d. a hexagon?
If so, describe how it can be done.

- For eight linked cubes, find the solids with the smallest and the largest surface area.
  Draw the shapes on isometric paper.
  Extend to 12 cubes.

- *Imagine you have a cube.*
  Put a dot in the centre of each face.
  Join the dots on adjacent sides by straight lines.
  What shape is generated by these lines?

- *Visualise an octahedron.*
  Put a dot in the centre of each face.
  Join the dots on adjacent sides by straight lines.
  What shape is generated by these lines?

- *Imagine a slice cut symmetrically off each corner of a cube. Describe the solid which remains.*
  Is there more than one possibility?
  Repeat for a tetrahedron or octahedron.

- *Triangles are made by joining three of the vertices of a cube. How many different-shaped triangles can you make like this?*
  Draw sketches of them.

Link to plane symmetry (pages 206–7).