Pupils should be taught to:

Consolidate understanding of the operations of multiplication and division, their relationship to each other and to addition and subtraction; know how to use the laws of arithmetic

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
operation, commutative, inverse, add, subtract, multiply, divide, sum, total, difference, product, multiple, factor, quotient, divisor, remainder…

Understand addition, subtraction, multiplication and division as they apply to whole numbers and decimals.

Multiplication

Understand that:
- Multiplication is equivalent to and is more efficient than repeated addition.
- Because multiplication involves fewer calculations than addition, it is likely to be carried out more accurately.

Compare methods and accuracy in examples such as:
- Find the cost of 38 items at £1.99 each.

Conclude it is easier to calculate £2 × 38 then compensate by 38p than to add £1.99 a total of 38 times, or calculate 1.99 × 38.

Understand the effect of multiplying by 0 and 1.

Division

Recognise that:
- 910 ÷ 13 can be interpreted as ‘How many 13s in 910?’, and calculated by repeatedly subtracting 13 from 910, or convenient multiples of 13.
- Division by 0 is not allowed.
- A quotient (the result obtained after division) can be expressed as a remainder, a fraction or as a decimal, e.g.
  90 ÷ 13 = 6 R 12
  or 90 ÷ 13 = $6\frac{12}{13}$
  or 90 ÷ 13 = 6.92 (rounded to two decimal places)
  The context often determines which of these is most appropriate.

Decide in the context of a problem how to express and interpret a quotient – that is:
- whether to express it with a remainder, or as a fraction, or as a decimal;
- whether to round it up or down;
- what degree of accuracy is required.

For example:
- Four small cars cost a total of £48 623. What should a newspaper quote as a typical cost of a small car?
  An appropriate answer is rounded: about £12 000 each.

- 107 pupils and staff need to be taken to the theatre. How many 15-seater minibuses should be ordered?
  $7\frac{2}{15}$ minibuses is not an appropriate answer for this example. To round $7\frac{2}{15}$ down to 7 would leave 2 people without transport. 8 minibuses is the appropriate answer.

- How many boxes of 60 nails can be filled with 340 nails?
  340 ÷ 60 = 5 R 40 or $5\frac{2}{3}$, but the appropriate answer is obtained by rounding down to 5, ignoring the remainder.

See Y456 examples (pages 52–7).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to: *associative, distributive… partition…*

Understand the operations of addition, subtraction, multiplication and division as they apply to positive and negative numbers.

Link to integers (pages 48–51).

Understand the operations of addition and subtraction as they apply to fractions.

Link to fractions (pages 66–9).

Understand that multiplying does not always make a number larger and that division does not always make a number smaller.

Recognise that:

- 9.1 ÷ 0.1 can be interpreted as 'How many 0.1s (or tenths) in 9.1?'
- 9.1 ÷ 0.01 can be interpreted as 'How many 0.01s (or hundredths) in 9.1?'

Link to multiplying and dividing by 0.1 and 0.01 (pages 38–9).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to: *reciprocal…*

Understand the effect of multiplying and dividing by numbers between 0 and 1.

Understand the operations of multiplication and division as they apply to fractions.

Link to fractions (pages 66–9).

Understand that multiplying a positive number by a number between 0 and 1 makes it smaller and that dividing it by a number between 0 and 1 makes it larger. Use this to check calculations and to estimate the order of magnitude of an answer.

Generalise inequalities such as:
if $p > 1$ and $q > 1$, then $pq > p$.

Know the effect on inequalities of multiplying and dividing each side by the same negative number.

Know and understand that division by zero has no meaning. For example, explore dividing a number by successively smaller positive decimals approaching zero, then negative decimals approaching zero.

Link to multiplying and dividing by any integer power of 10 (pages 38–9), checking results (pages 110–11), and inequalities (pages 130–1).

Recognise and use reciprocals. *Know that:*

- *A number multiplied by its reciprocal equals 1, e.g. the reciprocal of 4 is ¼ and of 7 is ⅐.*
- *The reciprocal of a reciprocal gives the original number.*

Find the reciprocal of a number and use the reciprocal key on a ***calculator****, recognising that the answer may be inexact. For example:*

- *What is the reciprocal of:*
  a. *0.3* b. *27* c. *0.0027?*
- *A ∗ stands in the place of any missing digit.*
  The reciprocal of a whole number between 0 and 100 is 0.02∗26, to four significant figures.
  Find the number, and the missing digit.
- *The reciprocal of a whole number between 100 and 1000 is 0.0012∗∗5, to five significant figures.*
  Find the number, and the missing digits.

Link to reciprocal function sequences (pages 108–9).

Pupils should be taught to:

Consolidate understanding of the operations of multiplication and division, their relationship to each other and to addition and subtraction; know how to use the laws of arithmetic (continued)

As outcomes, Year 7 pupils should, for example:

When dividing using a **calculator**, interpret the quotient in the context of a problem involving money, metric measures or time.

3.05

For example, depending on the context:
- A display of '3.05' could mean £3.05, 3 kilograms and 50 grams, or 3 hours and 3 minutes.
- A display of '5.2' could mean £5.20, 5 metres and 20 centimetres, or 5 hours and 12 minutes.

Relate division to fractions. Understand that:
- $\frac{1}{4}$ of 3.6 is equivalent to 3.6 ÷ 4.
- 7 ÷ 8 is equivalent to $\frac{7}{8}$.
- $\frac{50}{3}$ is equivalent to 50 ÷ 3.

See Y456 examples (pages 54–7).

Link to finding fractions of numbers (pages 66–7).

Know how to use the **laws of arithmetic** to support efficient and accurate mental and written calculations, and calculations with a **calculator**.

Examples of commutative law

$4 \times 7 \times 5 = 4 \times 5 \times 7 = 20 \times 7 = 140$
or $\quad = 7 \times 5 \times 4 = 35 \times 4 = 140$

To find the area of a triangle, base 5 cm and height 6 cm:
area = $\frac{1}{2} \times 5 \times 6 = \frac{1}{2} \times 6 \times 5 = 3 \times 5 = 15$ cm^2

Example of associative law

$15 \times 33 = (5 \times 3) \times 33$ or $5 \times (3 \times 33) = 5 \times 99 = 495$

Example of distributive law

$3.7 \times 99 = 3.7 \times (100 - 1)$
$= (3.7 \times 100) - (3.7 \times 1)$
$= 370 - 3.7$
$= 366.3$

Link to algebraic operations (pages 114–17), and mental calculations (pages 92–7).

Inverses

Understand that addition is the inverse of subtraction, and multiplication is the inverse of division. For example:
- Put a number in a **calculator**. Add 472 (or multiply by 26). What single operation will get you back to your starting number?
- Fill in the missing number: (□ × 4) ÷ 8 = 5.

Use inverses to check results. For example:
- 703 ÷ 19 = 37 appears to be about right, because 36 × 20 = 720.

Link to inverse operations in algebra (pages 114–15), and checking results (pages 110–11).

As outcomes, Year 8 pupils should, for example:

Continue to use the **laws of arithmetic** to support efficient and accurate mental and written calculations, and calculations with a **calculator**.

For example, use mental or informal written methods to calculate:

- $484 \times 25 = 484 \times 100 \div 4 = 48\,400 \div 4 = 12\,100$
- $3.15 \times 25 = 3.15 \times 100 \div 4 = 315 \div 4 = 78.75$
- $28 \times 5 = 28 \times 10 \div 2 = 280 \div 2 = 140$
- $15 \times 8 = 15 \times 2 \times 2 \times 2 = 120$
- $6785 \div 25 = 6785 \div 5 \div 5 = 1357 \div 5 = 271.4$

Recognise the application of the distributive law when multiplying a single term over a bracket in number and in algebra.

Link to algebraic operations (pages 114–17), and mental calculations (pages 92–7).

Inverses
Use inverse operations. For example:
- Put a number in your **calculator**.
  Add 46, then multiply by 17.
  What must you do to get back to the starting number?
- Fill in the missing number:
  $\square^2 \div 4 = 16$

Use inverses to check results. For example:
- $6603 \div 18.6 = 355$ appears to be about right, because $350 \times 20 = 7000$.

Link to inverse operations in algebra (pages 114–15), and checking results (pages 110–11).

As outcomes, Year 9 pupils should, for example:

Continue to use the **laws of arithmetic** to support efficient and accurate mental and written calculations, and calculations with a **calculator**.

Recognise the application of the distributive law when expanding the product of two linear expressions in algebra.

Link to algebraic operations (pages 114–21), and mental calculations (pages 92–7).

Inverses
Use inverse operations. For example:
- Put a number in your **calculator**. Cube it. What must you do to get back to the starting number?
- Put a number in your **calculator**. Find the square root. What must you do to get back to the starting number?

Explain why it may not be possible to get back exactly to the starting numbers using a calculator.

Use inverses to check results of calculations.

Link to algebraic operations (pages 114–21), and checking results (pages 110–11).

Pupils should be taught to:

Know and use the order of operations, including brackets

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
order of operations, brackets…

Know the conventions that apply when evaluating expressions:
- Contents of brackets are evaluated first.
- In the absence of brackets, multiplication and division take precedence over subtraction and addition.
- A horizontal line acts as a bracket in expressions such as $\frac{5 + 6}{2}$ or $\frac{a + b}{5}$.

brackets
⇩
powers or indices
⇩
multiplication (including 'of') and division
⇩
addition and subtraction

- With strings of multiplications and divisions, or strings of additions and subtractions, and no brackets, the convention is to work from left to right, e.g. $12 \div 4 \div 2 = 1.5$, not 6.

Calculate with mixed operations. For example:

- Find mentally or use jottings to find the value of:
  a. $16 \div 4 + 8 = 12$
  b. $16 + 8 \div 4 = 18$
  c. $14 \times 7 + 8 \times 11 = 186$
  d. $\frac{100}{4 \times 5} = 5$
  e. $32 + 13 \times 5 = 97$
  f. $(3^2 + 4^2)^2 = 625$
  g. $(5^2 - 7) / (2^2 - 1) = 6$
- Use a **calculator** to calculate with mixed operations, e.g. $(32 + 13) \times (36 - 5) = 1395$
- In algebra recognise that, for example, when $a = 4$, $3a^2 = 3 \times 4^2 = 3 \times 16 = 48$

Link to calculator methods (pages 108–9), order of algebraic operations (pages 114–15), and substitution in expressions and formulae (pages 138–41).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

Recognise that, for example:

$$\frac{100}{4 \times 5} = 100 \div 4 \div 5 = 5$$

or $$\frac{a}{b \times c} = a \div (b \times c) \text{ or } a \div b \div c$$

Calculate with more complex mixed operations, including using the bracket keys on a **calculator**. For example:

- Find the value of:
  a. $2.1 - (3.5 + 2.1) + (5 + 3.5) = 5$
  b. $\frac{(2+3)^2}{(14-9)^2} = \frac{5^2}{5^2} = 1$

- Find, to two decimal places, the value of:
  a. $(5.5 + 2) / 7 = 1.07$ to 2 d.p.
  b. $\frac{8+4}{13-2} = 1.09$ to 2 d.p.
  c. $\frac{25}{6 \times 93} = 0.04$ to 2 d.p.
  d. $\sqrt{(26^2 - 14^2)} = 21.91$ to 2 d.p.

Evaluate expressions using nested brackets, such as:
$120 \div \{30 - (2 - 7)\}$

Understand that the position of the brackets is important. For example:
- Make as many different answers as possible by putting brackets into the expression
  $3 \times 5 + 3 - 2 \times 7 + 1$
  For example:
  a. $3 \times (5 + 3) - (2 \times 7) + 1 = 11$
  b. $3 \times (5 + 3) - 2 \times (7 + 1) = 8$
  c. $(3 \times 5) + 3 - (2 \times 7) + 1 = 5$
  d. $(3 \times 5) + (3 - 2) \times 7 + 1 = 23$
  e. $(3 \times 5) + (3 - 2) \times (7 + 1) = 23$
  f. $(3 \times 5) + 3 - (2 \times 7 + 1) = 3$

Link to calculator methods (pages 108–9), order of algebraic operations (pages 114–15), substitution in expressions and formulae (pages 138–41).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years.

Understand the effect of powers when evaluating an expression. For example:

- Find the value of:
  a. $36 \div (3 + 9) - 7 + 3 \times (8 \div 2)^3 = 188$
  b. $\frac{7 \times 8^2}{7 \times 2} = \frac{8^2}{2} = 32$
  c. $\frac{(7 \times 8)^2}{7 \times 2} = \frac{7 \times 8 \times 7 \times 8}{7 \times 2} = 7 \times 8 \times 4 = 224$
  d. $-7^2 + 5 = {}^{-}44$
  e. $({}^{-}7)^2 + 5 = 54$
  f. $(4/3)^2 = 4^2 \div 3^2 = 16/9 = 1\,7/9$

Calculate with more complex mixed operations, including using the bracket keys on a **calculator**. For example:

- Find the value of:
  $-(251 \times 3 + 281) + 3 \times 251 - (1 - 281) = {}^{-}1$

- Find, to two decimal places, the value of:
  a. $\frac{(12-5)^2(7-3)^2}{(8-5)^3} = \frac{7^2 \times 4^2}{3^3} = 29.04$ to 2 d.p.
  b. $\frac{(16-9)^2(17-15)^2}{3(16-11)^3} = \frac{7^2 \times 2^2}{3 \times 5^3} = 0.52$ to 2 d.p.

- In algebra recognise that when $a = 2$,
  a. $3a^2 - 9 = 3(2^2) - 9 = 3$
  b. $3(a^2 - 9) = 3(4 - 9) = {}^{-}15$
  c. $(3a)^2 - 9 = 6^2 - 9 = 27$

  Recognise that $({}^{-}a)^2 \neq -a^2$.

Link to calculator methods (pages 108–9), order of algebraic operations (pages 114–15), substitution in expressions and formulae (pages 138–41).

Pupils should be taught to:

Consolidate the rapid recall of number facts and use known facts to derive unknown facts

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
increase, decrease, double, halve, complement, partition…

Addition and subtraction facts

Know with rapid recall addition and subtraction facts to 20.

Complements

Derive quickly:

- whole-number complements in 100 and 50,
  e.g. 100 = 63 + 37, 50 = ⁻17 + 67
- decimal complements in 1 (one or two decimal places),
  e.g. 1 = 0.8 + 0.2, 1 = 0.41 + 0.59

Doubles and halves

Derive quickly:

- doubles of two-digit numbers including decimals,
  e.g. 23 × 2, 3.8 × 2, 0.76 × 2
- doubles of multiples of 10 to 1000,
  e.g. 670 × 2, 830 × 2
- doubles of multiples of 100 to 10 000,
  e.g. 1700 × 2, 6500 × 2

and all the corresponding halves.

Multiplication and division facts

Know with rapid recall multiplication facts up to 10 × 10, and squares to at least 12 × 12.
Derive quickly the associated division facts, e.g. 56 ÷ 7, √81.

Use knowledge of place value to multiply and divide mentally any number by 10, 100, 1000, or by a small multiple of 10.
For example:

- 4.3 × 100
- 60 ÷ 1000
- 1.6 × 20 = 16 × 2 = 32
- ☐ ÷ 100 = 4.7

Use knowledge of multiplication facts and place value to multiply mentally examples such as:

- 0.2 × 8
- 0.04 × 9
- 8 × 0.5
- 7 × 0.03
- ☐ × 0.2 = 10
- 80 × ☐ = 8

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

Use known facts to derive unknown facts
For example, generate constant-step sequences, such as:
- Start at 108, the rule is 'add 8'.
- The start number is 5, target is 33. What is the rule?

Complements
Derive quickly:
- complements in 1, 10, 50, 100, 1000.

Solve mentally equations such as:
- $100 = x + 37$
- $10 = 3.62 + x$
- $50 - x = 28$
- $220 = 1000 - x$

Doubles and halves
Use doubling and halving methods to multiply and divide by powers of 2. For example:
- $18 \times 16 = 18 \times 2 \times 2 \times 2 \times 2$
- $180 \div 8 = 180 \div 2 \div 2 \div 2$

Link to using the laws of arithmetic (pages 84–5).

Multiplication and division facts
Derive the product and quotient of multiples of 10 and 100 (whole-number answers). For example:
- 30×60
- $1400 \div 700$
- 900×20
- $6300 \div 30$

Use knowledge of place value to multiply and divide whole numbers by 0.1 and 0.01. For example:
- 47×0.1
- $8 \div 0.1$
- 9×0.01
- $16 \div 0.1$
- 432×0.01
- $37 \div 0.01$

Extend to decimals, such as:
- 0.5×0.1
- $5.2 \div 0.01$
- $0.1 \times \square = 0.08$
- $\square \div 0.01 = 3$

Use knowledge of multiplication and division facts and place value to:
derive products involving numbers such as 0.4 and 0.04. For example:
- $4 \times 0.6 \quad = 4 \times 6 \div 10 \quad = 24 \div 10 = 2.4$
- $0.7 \times 0.9 \quad = 7 \times 9 \div 100 \quad = 0.63$
- $0.04 \times 8 \quad = 4 \times 8 \div 100 \quad = 0.32$
- $\square \times \square \quad = 0.08$

divide mentally by 2, 4 and 5. For example:
- $0.2 \div 4 \quad = 2 \div 4 \div 10 \quad = 0.5 \div 10 = 0.05$
- $0.03 \div 5 \quad = 3 \div 5 \div 100 \quad = 0.6 \div 100 = 0.006$
- $\square \div \square = 0.4$

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years.

Use known facts to derive unknown facts
For example:
- Derive 36×24 from 36×25.

Multiplication and division facts
Derive the product and quotient of multiples of 10, 100 and 1000. For example:
- 600×7000
- $400 \div 8000$
- $48\,000 \div 800$
- $60 \div 90\,000$

Use knowledge of place value to multiply and divide decimals by 0.1 and 0.01. For example:
- 0.47×0.1
- $0.8 \div 0.1$
- 9.6×0.01
- $0.016 \div 0.1$
- 0.0432×0.01
- $3.7 \div 0.01$
- $0.01 \times \square = 1.7$
- $\square \div 0.01 = 3.2$

Consolidate knowledge of multiplication and division facts and place value to multiply and divide mentally.
For example:
- $0.24 \times 0.4 = 24 \times 4 \div 1000 = 96 \div 1000 = 0.096$
- $800 \times 0.7 = 80 \times 7 = 56 \times 10 = 560$
- $72 \div 0.9 = 72 \div 9 \times 10 = 8 \times 10 = 80$
- $0.48 \div 0.6 = 4.8 \div 6 = 48 \div 6 \div 10 = 8 \div 10 = 0.8$
- $720 \div 0.03 = 72\,000 \div 3 = 24\,000$
- $\square \times \square \times \square = 0.08$

Link to using the laws of arithmetic (pages 84–5).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Consolidate the rapid recall of number facts and use known facts to derive unknown facts (continued)

Factors, powers and roots
Know or derive quickly:
- prime numbers less than 30;
- squares of numbers 0.1 to 0.9, and of multiples of 10 to 100, and the corresponding roots;
- pairs of factors of numbers to 100.

Calculate mentally:
- $4^2 + 9$
- $(4 + 3)^2$
- $4^2 + 5^2$
- $5^2 – 7$
- $\sqrt{(9 + 7)}$
- $\sqrt{(40 – 2^2)}$
- What is the fourth square number?

Solve mentally:
- $3a = 15$
- $x^2 = 49$
- $n(n + 1) = 12$

Link to multiples, factors and primes (pages 52–5), and powers and roots (pages 56–9).

Fraction, decimal and percentage facts
See pages 70–1.

Measurements
Recall and use formulae for:
- the perimeter and area of a rectangle.

Calculate simple examples mentally.

Recall:
- relationships between units of time;
- relationships between metric units of length, mass and capacity (e.g. between km, m, cm and mm).

Convert between units of measurement. For example:
- Convert 38 cm into mm.
- Convert 348p into pounds.
- Convert 45 minutes into seconds.

See Y456 examples (pages 38–9, 58–9, 90–3).

Link to measures and mensuration (pages 228–31).

As outcomes, Year 8 pupils should, for example:

Factors, powers and roots

Know or derive quickly:

- cubes of numbers from 1 to 5, and 10, and the corresponding roots;
- the prime factorisation of numbers to 30.

Calculate mentally:

- $\sqrt{(24 + 12)}$
- $(7 + 4)^2$
- $\sqrt{(89 - 25)}$
- $(12 + 9 - 18)^2$

Solve mentally:

- $3a - 2 = 31$
- $n(n - 1) = 56$

Link to multiples, factors and primes (pages 52–5), and powers and roots (pages 56–9).

Fraction, decimal and percentage facts

See pages 70–1.

Measurements

Recall and use formulae for:

- the perimeter and area of a rectangle;
- the area of a triangle;
- the volume of a cuboid.

Calculate simple examples mentally.

Know and use rough metric equivalents for:
1 mile, 1 yard, 1 pound (lb), 1 gallon, 1 pint,
and rough imperial equivalents for:
1 km, 1 m, 1 kg, 1 litre.

For example, use 5 miles ≈ 8 kilometres to work out:

- The signpost said that it was 50 miles to London. How many kilometres is that, approximately?
- The jogger was pleased that she had run 32 km. About how many miles is this?

Convert between units of time. For example:

- How many minutes in:
  3 hours, 4.5 hours, 2.25 hours, 5 hours 25 minutes?
- Change to hours and minutes:
  120 minutes, 75 minutes, 300 minutes.
- How many hours in:
  3 days, 5¼ days, 1 week 2 days, …?
- How many days in:
  36 hours, 100 hours, the last 3 months of the year?
- How many days to Christmas? Your birthday?

Link to measures and mensuration (pages 228–31).

As outcomes, Year 9 pupils should, for example:

Factors, powers and roots

Find mentally:

- the HCF and LCM of pairs of numbers such as
  36 and 48, 27 and 36;
- products of small integer powers, such as
  $3^3 \times 4^2 = 27 \times 16 = 432$;
- factor pairs for a given number.

Calculate mentally:

- $(23 - 15 + 4 - 8)^3$
- $\sqrt[3]{(89 + 36)}$

Solve mentally:

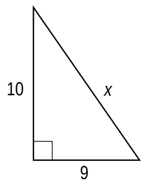
- $(3 + x)^2 = 25$
- $(12 - x)^2 = 49$

Identify numbers from property questions, such as:

- This number is a multiple of 5.
  It leaves remainder 1 when divided by 4.
  What could it be?
- This number has a digit sum of 6. It is divisible by 7.
  What is it?

Know simple Pythagorean triples such as 3, 4, 5, or 5, 12, 13, and their multiples.

- *Apply Pythagoras' theorem:*
  $x^2 = 9^2 + 10^2 = 181$
  $x = \sqrt{181}$



Measurements

Recall and use formulae for:

- the perimeter of a rectangle and circumference of a circle;
- the area of a rectangle, triangle, parallelogram, trapezium, circle;
- the volume of a cuboid and a prism.

Calculate simple examples mentally.

Link to measures and mensuration (pages 228–31), and use of compound measures in science.

Know that speed = distance/time.
Use this to derive facts from statements such as:

- *A girl takes 20 minutes to walk to school, a distance of 1.5 km.*
  Find her average speed in km/h.

Solve problems such as:

- *£1 is equivalent to 1.65 euros.*
  £1 is also equivalent to 1.5 US dollars ($1.5).
  How many euros are equivalent to $6?
- *A car travels 450 km on 50 litres of fuel.*
  How many litres of fuel will it use to travel 81 km?

Link to speed and solving problems involving constant rates of change (pages 232–3).

Pupils should be taught to:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings

As outcomes, Year 7 pupils should, for example:

Strategies for mental addition and subtraction

Count forwards and backwards from any number.
For example:
- Count on in 0.1s from 4.5.
- Count back from 4.05 in 0.01s.
- Count on from and back to zero in steps of ¾.

Identify positions of 0.1s and 0.01s on a number line.

Use a **spreadsheet** to replicate cells, e.g. to 'count' from 1 in steps of 1.

	A	B	C	D	E	F	G
1	1	= A1+1	= B1+1	= C1+1	= D1+1	= E1+1	= F1+1

	A	B	C	D	E	F	G
1	1	2	3	4	5	6	7

Modify the spreadsheet to count from 0.5 in steps of 0.1.

Add and subtract several small numbers.
For example:
- 4 + 8 + 12 + 6 + 13
- 5 – 4 + 8 – 10 – 7

Extend to adding and subtracting several small multiples of 10:
- 40 + 30 + 20
- 60 + 50 – 30

Continue to add and subtract any pair of two-digit whole numbers, such as 76 + 58, 91 – 47.
Extend to:
- adding and subtracting a two-digit whole number to or from a three-digit whole number;
- adding and subtracting decimals such as:

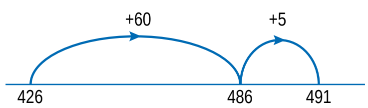
  8.6 ± 5.7   0.76 ± 0.58   0.82 ± 1.5

  by considering

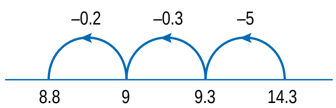
  86 ± 57   76 ± 58   82 ± 150

Use jottings such as an empty number line to support or explain methods for adding and subtracting mentally. Choose an appropriate method, such as one of the following:

Partition and deal with the most significant digits first.
For example:
- 426 + 65 = (426 + 60) + 5 = 486 + 5 = 491



- 14.3 – 5.5 = 14.3 – 5 – 0.3 – 0.2 = 9 – 0.2 = 8.8



As outcomes, Year 8 pupils should, for example:

Strategies for mental addition and subtraction

Consolidate and use addition and subtraction strategies from previous years. For example:

Add and subtract mentally pairs of integers.
Use strategies for addition and subtraction to add and subtract pairs of integers. For example:

- $^{-}3 + {}^{-}5 = \ldots$ $\quad$ $^{-}13 + {}^{-}25 = \ldots$
  $^{-}46 + {}^{-}59 = \ldots$ $\quad$ $^{-}100 + {}^{-}99 = \ldots$
  $^{-}9 - {}^{-}14 = \ldots$ $\quad$ $^{-}43 - {}^{-}21 = \ldots$
  $^{-}37 - {}^{-}25 = \ldots$ $\quad$ $^{-}7 - {}^{-}7 = \ldots$

- The result of subtracting one integer from another is $^{-}29$.
  What could the integers be?

- $\square + \square = {}^{-}46$

Add mentally several positive or negative numbers, including larger multiples of 10. For example:
- $5 + {}^{-}4 + 8 + {}^{-}10 + {}^{-}7$
- $250 + 120 - 190$

Calculate a mean using an assumed mean.
For example:
- Find the mean of 18.7, 18.4, 19.1, 18.3 and 19.5.
  Use 19.0 as the assumed mean.
  The differences are $^{-}0.3$, $^{-}0.6$, 0.1, $^{-}0.7$ and 0.5, giving a total difference of $^{-}1.0$.
  The actual mean is $19.0 - (1.0 \div 5) = 18.8$.

Link to integers (pages 48–9).

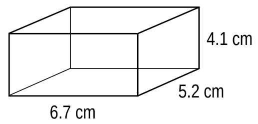
Add and subtract pairs of numbers of the same order (both with two significant figures). For example:
- $360 + 250$
- $4800 - 1900$
- $7.8 + 9.3$
- $0.081 - 0.056$

As outcomes, Year 9 pupils should, for example:

Strategies for mental addition and subtraction

Consolidate and use addition and subtraction strategies from previous years. For example:

- Find the length of wire in this framework.



$4(6.7) + 4(5.2) + 4(4.1) = 4 \times 16 = 64$ cm

Pupils should be taught to:

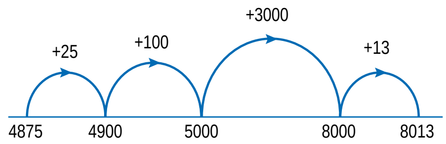
Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

As outcomes, Year 7 pupils should, for example:

Mental addition and subtraction strategies (continued)

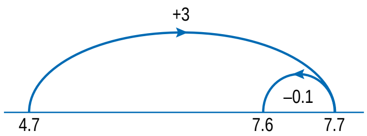
Find a difference by counting up from the smaller to the larger number. For example:

- 8013 – 4875 = 25 + 100 + 3000 + 13 = 3138

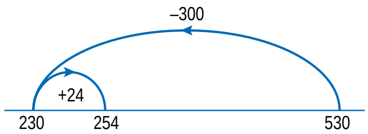


Use compensation, by adding or subtracting too much, and then compensating. For example:

- 4.7 + 2.9 = 4.7 + 3 – 0.1 = 7.7 – 0.1 = 7.6



- 530 – 276 = 530 – 300 + 24 = 230 + 24 = 254



Recognise special cases. For example:

Near doubles
- 8.5 + 8.2 = 16.7 (double 8.2 plus 0.3)
- 427 + 366 = 793 (double 400 plus 27 minus 34)

'Nearly' numbers
Add and subtract near 10s and near 100s, by adding or subtracting a multiple of 10 or 100 and adjusting. For example:

- 48 + 39
- 92 + 51
- 76 + 88
- 427 + 103
- 586 + 278
- 84 – 29
- 70 – 51
- 113 – 78
- 925 – 402
- 350 – 289

Use the relationship between addition and subtraction.
For example, recognise that knowing one of:

2.4 + 5.8 = 8.2   5.8 + 2.4 = 8.2
8.2 – 5.8 = 2.4   8.2 – 2.4 = 5.8

means that you also know the other three.

See Y456 examples (pages 40–7).

As outcomes, Year 8 pupils should, for example:	As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

Strategies for multiplication and division

Use factors. For example:

- 3.2 × 30 — 3.2 × 10 = 32; 32 × 3 = 96
- 156 ÷ 6 — 156 ÷ 3 = 52; 52 ÷ 2 = 26

Use partitioning. For example:
For multiplication, partition either part of the product:

- 7.3 × 11 = (7.3 × 10) + 7.3 = 73 + 7.3 = 80.3

For division, partition the dividend (the number that is to be divided by another):

- 430 ÷ 13 — 400 ÷ 13 = 30 R 10; 30 ÷ 13 = 2 R 4; 430 ÷ 13 = 32 R 14 = 33 R 1

Recognise special cases where doubling or halving can be used. For example:

To multiply by 50, first multiply by 100 and then divide by 2.
For example:

- 1.38 × 50 — 1.38 × 100 = 138; 138 ÷ 2 = 69

Double one number and halve the other. For example:

- 6 × 4.5 — 3 × 9 = 27
  12 × 7.5 — 6 × 15 = 3 × 30 = 90

Use the relationship between multiplication and division.
For example, knowing one of these facts means you also know the other three:

2.4 × 3 = 7.2	3 × 2.4 = 7.2
7.2 ÷ 3 = 2.4	7.2 ÷ 2.4 = 3

See Y456 examples (pages 60–5).

As outcomes, Year 8 pupils should, for example:

Strategies for multiplication and division

Use factors. For example:

- 22×0.02 — $22 \times 0.01 = 0.22$; $0.22 \times 2 = 0.44$
- $420 \div 15$ — $420 \div 5 = 84$; $84 \div 3 = 28$
- $126 \div 18$ — $126 \div 6 = 21$; $21 \div 3 = 7$

Use partitioning. For example, for multiplication, partition either part of the product:

- $13 \times 1.4 = (10 \times 1.4) + (3 \times 1.4) = 14 + 4.2 = 18.2$
- $7.3 \times 21 = (7.3 \times 20) + 7.3 = 146 + 7.3 = 153.3$

Use knowledge of place value to multiply and divide mentally any number by 0.1 or 0.01. For example:

- 3.6×0.1 — $3.6 \div 10$
- 99.2×0.01 — $99.2 \div 100$
- $^{-}1.8 \div 0.1$ — $^{-}1.8 \times 10$
- $0.35 \div 0.01$ — 0.35×100

$$\begin{array}{ccccc} 99.2 & \times & 0.01 & & 0.992 \\ \downarrow \times 10 & & \downarrow \times 100 & & \uparrow \div 1000 \\ 992 & \times & 1 & = & 992 \end{array}$$

Recognise special cases where doubling or halving can be used. For example:

Extend doubling and halving methods to include decimals and negative numbers. For example:

- 3.4×4.5 — $1.7 \times 9 = 15.3$
- 8.12×2.5 — $4.06 \times 5 = 20.3$
- 22×3.01 — $11 \times 6.02 = 66.22$
- $^{-}17 \times 1.5$ — $^{-}8.5 \times 3 = {}^{-}25.5$
- $^{-}8.4 \times {}^{-}1.25$ — $^{-}4.2 \times {}^{-}2.5 = {}^{-}2.1 \times {}^{-}5 = 10.5$

Multiply by near 10s.
For example:

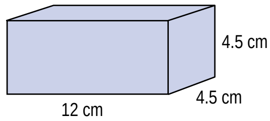
- $23 \times 11 = 230 \times 10 + 23 = 253$
- $75 \times 29 = 75 \times 30 - 75 = 2175$
- $8 \times {}^{-}19 = 8 \times ({}^{-}20 + 1) = {}^{-}160 + 8 = {}^{-}152$

As outcomes, Year 9 pupils should, for example:

Strategies for multiplication and division

Consolidate and use multiplication and division strategies from previous years. For example:

- Find the volume of this square-based cuboid.



$$4.5 \times 4.5 \times 12 = \frac{9}{2} \times \frac{9}{2} \times 12 = 9 \times 9 \times 3 = 243\text{ cm}^3$$

or

$$4.5 \times 4.5 \times 12 = (4.5 \times 12) \times 4.5 = 54 \times 4.5 = 216 + 27 = 243\text{ cm}^3$$

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

Recall of fraction, decimal and percentage facts

Know or derive quickly:
- simple decimal/fraction/percentage equivalents, such as:
  - $\frac{1}{4}$ = 0.25 or 25% — 0.23 is equivalent to 23%
  - $\frac{7}{10}$ = 0.7 or 70% — 57% is equivalent to 0.57 or $\frac{57}{100}$
- simple addition facts for fractions, such as:
  - $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ — $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
- some simple equivalent fractions for $\frac{1}{4}$ and $\frac{1}{2}$, such as:
  - $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{50}{100}$
  - $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{25}{100}$

Strategies for finding equivalent fractions, decimals and percentages

For example:
- Convert $\frac{1}{8}$ into a decimal.
  (Know that $\frac{1}{4}$ = 0.25 so $\frac{1}{8}$ is 0.25 ÷ 2 = 0.125.)
- Express $\frac{3}{5}$ as a percentage.
  (Know that $\frac{3}{5} = \frac{6}{10}$ or $\frac{60}{100}$, so it is equivalent to 60%.)
- Express 23% as a fraction and as a decimal.
  (Know that 23% is equivalent to $\frac{23}{100}$ or 0.23.)
- Express 70% as a fraction in its lowest terms.
  (Know that 70% is equivalent to $\frac{70}{100}$, and cancel this to $\frac{7}{10}$.)

Use known facts such as $\frac{1}{5}$ = 0.2 to convert fractions to decimals mentally. For example:

$\frac{3}{5}$ = 0.2 × 3 = 0.6

Find simple equivalent fractions.
For example:
- State three fractions equivalent to $\frac{3}{5}$, such as:
  $\frac{6}{10}$, $\frac{30}{50}$, $\frac{24}{40}$
- Fill in the boxes:
  $\frac{3}{4} = \frac{\square}{8} = \frac{\square}{12} = \frac{\square}{16} = \frac{\square}{20}$
  $\frac{7}{\square} = \frac{21}{30}$

Strategies for calculating fractions and percentages of whole numbers and quantities. For example:
- $\frac{1}{8}$ of 20 = 2.5 (e.g. find one quarter, halve it)
- 75% of 24 = 18 (e.g. find 50% then 25% and add the results)
- 15% of 40 (e.g. find 10% then 5% and add the results)
- 40% of 400 kg (e.g. find 10% then multiply by 4)

- 60 pupils go to the gym club.
  25% of them are girls.
  How many are boys?

See Y456 examples (pages 24–5, 32–3).

Link to finding fractions and percentages of quantities (pages 66–7, 72–3).

As outcomes, Year 8 pupils should, for example:

Recall of fraction, decimal and percentage facts

Know or derive quickly:

- decimal/fraction/percentage equivalents such as:
  $\frac{1}{8}$ = 0.125 or 12½% $\frac{3}{5}$ = 0.6 or 60%
  1¾ = 1.75 or 175% $\frac{1}{3} \approx$ 0.33 or 33⅓%
- the simplified form of fractions such as:
  $\frac{3}{15} = \frac{1}{5}$ $\frac{14}{21} = \frac{2}{3}$

Know that ⅓ is $0.\dot{3}$ and ⅔ is $0.\dot{6}$.

Know that 0.03 is $\frac{3}{100}$ or 3%.

Strategies for finding equivalent fractions, decimals and percentages

For example:

- Express 136% as a decimal.
  (Know that 136% is equivalent to $\frac{136}{100}$ or 1.36.)
- Express 55% as a fraction in its lowest terms.
  (Know 55% is equivalent to $\frac{55}{100}$, cancel to $\frac{11}{20}$.)
- Express $\frac{13}{20}$ as a percentage.
  (Work out that $\frac{13}{20} = \frac{65}{100}$, so it is equivalent to 65%.)
- Convert $\frac{4}{25}$ into a decimal.
  (Work out that $\frac{4}{25} = \frac{16}{100}$, so it is equivalent to 0.16.)
- Convert 0.45 into a fraction.
  (Know that 0.45 = $\frac{45}{100}$, and simplify this to $\frac{9}{20}$.)
- Express 0.06 as a percentage.
  (Recognise this as $\frac{6}{100}$ or 6%.)

Use known facts such as ⅛ = 0.125 to convert fractions to decimals mentally. For example:
$\frac{5}{8} = 0.125 \times 5 = 0.625$

Convert between improper fractions and mixed numbers. For example:

- Convert 7⅓ into an improper fraction.
- Convert $\frac{36}{5}$ into a mixed number.

Strategies for calculating fractions and percentages

of whole numbers and quantities. For example:

- $\frac{3}{5}$ of 20 = 12 (e.g. find one fifth, multiply by 3)
- 1½ of 16 = 24 (e.g. find one half, add it to 16)
- 125% of 240 (e.g. find 25%, add it to 240)
- 35% of 40 (e.g. find 10% then 30% then 5%, add the last two results)

- There is a discount of 5% on a £45 coat in a sale. By how much is the coat's price reduced?
  (e.g. 1% = 45p so 5% = (45 × 5)p = £2.25
  or 10% = £4.50 so 5% = £2.25)

Link to finding fractions and percentages of quantities (pages 66–7, 72–3).

As outcomes, Year 9 pupils should, for example:

Recall of fraction, decimal and percentage facts

Know that 0.005 is half of one per cent,
so that 37.5% = 37% + 0.5%
or 0.37 + 0.005 = 0.375

Strategies for finding equivalent fractions, decimals and percentages

For example:

- Express 0.625 as an equivalent percentage.
  (Recognise this as 62%, plus half of one per cent, or 62.5%.)
- Express 10.5 as an equivalent percentage.
  (Recognise this as 1000% plus 50%, or 1050%.)

Simplify fractions by cancelling highest common factors mentally. For example:

- Simplify:
  $\frac{85}{100}$ $\frac{630}{720}$

Strategies for calculating fractions and percentages

of numbers and quantities. For example:

- $\frac{2}{5}$ of 20.5 = 8.2
  (e.g. find one fifth, multiply by 2)
- $\frac{3}{8}$ of 6400 = 2400
  (e.g. find one eighth, multiply by 3)

- Find 20% of £3.50.
- Find 35% of £5.
- Increase 480 kg by 20%.
- Decrease 500 mm by 12%.
- 25% of a number is 12. What is the number?

Link to finding fractions and percentages of quantities (pages 66–7, 72–3).

Pupils should be taught to:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

As outcomes, Year 7 pupils should, for example:

Word problems and puzzles (all four operations)

Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

- Arrange the digits 3, 5 and 2 to make the largest possible odd number.
- Write in figures the number two and a quarter million.
- A girl scored 67 in her first innings and 128 in her second innings. What was her total score?
- Pencils cost 37p each.
  How many pencils can you buy with £3.70?
- A 55 g bag of crisps has 20% fat. How much fat is that?
- A boy saved £215. He bought a Walkman for £69.
  How much money did he have left?
- A girl used 2 metres of wood to make 5 identical shelves.
  How long was each shelf?
- Estimate the value of 51 × 19.
- Find two numbers whose sum is 14 and whose product is 48.
- There are 12 green buttons and 4 white buttons in a tin.
  I choose one button at random from the tin.
  What is the probability it is a white button?

Written questions

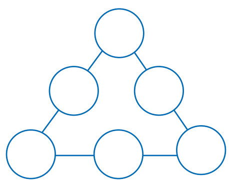
- Sandy and Michael dug a neighbour's garden.
  They were paid £32 to share for their hours of work.
  Sandy worked for 6 hours. Michael worked for 2 hours.
  How much should Sandy get paid?
- The mean of *a*, *b* and *c* is 6. *a* is 5 and *b* is 11. What is *c*?
- Tony, David and Estelle are playing a team game.
  They need to get a mean of 75 points to win.
  Tony scores 63 points, Estelle scores 77 points and David scores 77 points. Have they scored enough points to win?
- What is the value of $6n + 3$ when $n = 2.5$?

Solve problems or puzzles such as:

- Three consecutive integers add up to 87.
  What are they?

- Choose from 1, 2, 3, 4 and 5 to place in the boxes.
  In any question, you cannot use a number more than once.
  a. □ – □ + □ = 5
  b □ + □ – □ = 4
  c. □ × □ – □ = 3
  d. (□ + □) ÷ □ = 2
  e. (□ + □) ÷ (□ + □) = 1

- Use each of the numbers
  1, 2, 4, 6, 8, 12 once.
  Write one number in each circle.
  The product of the three numbers on each side of the triangle must be 48.



- Write any number up to 40.
  Multiply its last digit by 4 then add the other digit to this.
  Repeat the process until you get back to the original.
  What is the longest chain you can make?

As outcomes, Year 8 pupils should, for example:

Word problems and puzzles (all four operations)

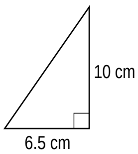
Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

- Write in figures the number that is one less than seven and a half million.
- Two angles fit together to make a straight line. One of them is 86°. What is the other?
- 1 ounce is about 28 grams. About how many grams are 3 ounces?
- Four oranges cost 37p. What do 12 oranges cost?
- How many metres are there in 2.5 kilometres?
- A person's heart beats 70 times in 1 minute. How many times does it beat in 1 hour?
- Carpet tiles are 50 cm by 50 cm. How many are needed to cover one square metre?
- Estimate the value of 502 ÷ 49.
- Solve 45 + x = 92.
- The probability that it will rain in August is 0.05. What is the probability it will not rain in August?

Written questions

- 14 × 39 = 546. What is 14 × 3.9?
- Four sunflowers have heights of 225 cm, 199 cm, 185 cm and 239 cm. What is their mean height?
- The sum of p and q is 12. The product of p and q is 27. Calculate the values of p and q.
- Find 25% of 10% of £400.
- Calculate the area of this triangle.



Solve problems or puzzles such as:

- Make 36 using any combination of +, –, ×, ÷, and brackets, and each of 1, 3, 3 and 5 once.
- The numbers 3 and 10 are written on the front of two cards. There is a different number on the back of each card. When the two cards are on the table, the sum of the two numbers showing is 12, 13, 14 or 15. What two numbers are on the back of the cards?
- Use each of the digits 1, 2, 3, 4, 5, 6, 7 once. Write them in the boxes to make this statement true:
  □□ + □□ + □□ + □ = 100
- Take a pair of consecutive integers. Square each of them. Find the difference of the two squares. Repeat with different pairs of consecutive integers. Repeat with a pair of numbers whose difference is 2, or 3, or 4 …

As outcomes, Year 9 pupils should, for example:

Word problems and puzzles (all four operations)

Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

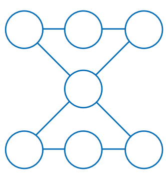
- Two years ago Jim's height was 1.48 metres. Now Jim's height is 1.7 metres. How much has Jim grown?
- Two of the angles of a triangle are 47° and 85°. Calculate the third angle.
- You get \$56 for £40. How many dollars do you get for £100?
- 75 miles per hour is about the same as 33 metres per second. About how many metres per second is 50 miles per hour?
- In a raffle, half of the tickets are bought by men. One third are bought by women. The rest are bought by children. What fraction of the tickets are bought by children?
- The ratio of men to women in a room is 3 to 5. There are 12 men. How many women are there?
- x = 2 and y = 3. Work out the value of x to the power y plus y to the power x.

Written questions

- The probability that a train will be late is 0.03. Of 50 trains, how many would you expect to be late?
- Find 1% of 2% of £1000.
- Some girls and boys have £32 between them. Each boy has £4 and each girl has £5. How many boys are there?

Solve problems or puzzles such as:

- You can use four 8s to make 10, e.g. (8 + 8)/8 + 8. Using any of +, –, ×, ÷ and brackets, and eight 8s, make the number 1000.
- Find two numbers:
  whose sum is 0.8 and whose product is 0.15;
  whose sum is $^{-}11$ and whose product is 28;
  whose difference is 4 and whose quotient is 3;
  whose difference is 2 and whose quotient is $^{-}1$.
- The product of two numbers is six times their sum. The sum of their squares is 325. What are the two numbers?
- Use each of the prime numbers 5, 7, 11, 13, 17, 19, 23 once. Write one in each circle so that the three primes in each line add up to the same prime number.



Pupils should be taught to:

Make and justify estimates and approximations (of numbers and calculations)

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
guess, estimate, approximate, roughly, nearly, approximately, too many, too few, enough, not enough… and the symbol ≈.

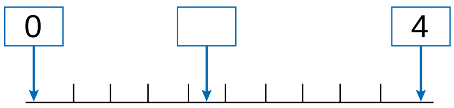
Understand that there are occasions when there is no need to calculate an exact answer and an estimate is sufficient.

Understand that the context affects the method used for estimating. For example:

- If I have £10 and some shopping to do, I need to round the amounts up in order to check I have enough money.
- If I estimate how much paint I need to paint a room, I need to round up so I have enough paint.

Estimate the position of a point on a marked scale, given the values of the end points. For example:

- Estimate the number that the arrow is pointing to:



if the end points of the scale are 0 and 4;
if the end points are ⁻5 and 5;
if the end points are 2.7 and 4.7.

Know that there are different ways for finding an approximate answer. For example:

- An approximate answer for 404 – 128 can be
  400 – 100 = 300 or 400 – 130 = 270
  Which is the better estimate?
- An approximate answer for 7.5 × 2.5 can be
  7 × 3 = 21
  or 8 × 2 = 16
  or between 7 × 2 = 14 and 8 × 3 = 24.
  Use a **calculator** to check which is the closer estimate.

Recognise what makes a 'good approximation'.
Answer questions such as:

- Which is the best approximation for 40.8 – 29.7?
  A. 408 – 297 C. 41 – 30
  B. 40 – 29 D. 4.0 – 2.9
- Which is the best approximation for 9.18 × 3.81?
  A. 10 × 3 C. 9 × 3
  B. 10 × 4 D. 9 × 4

See Y456 examples (pages 10–11).

Link to rounding (pages 42–5), and checking results of calculations (pages 110–11).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

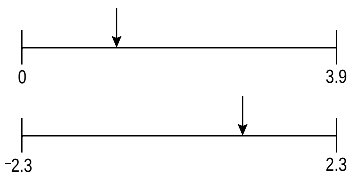
Begin to recognise when an exact answer is not needed and an estimate is sufficient, or when an exact answer is needed and an estimate is insufficient.

Estimate large numbers by estimating a small proportion then scaling up, in mathematics and other subjects. For example, estimate:

- the number of clover plants on a lawn;
- the daily volume of water a typical person drinks.

Estimate the position of a point on an unmarked scale. For example:

- Estimate the number that the arrow is pointing to.



Estimate squares and square roots. For example:

- Estimate $\sqrt{30}$.
  $\sqrt{25} < \sqrt{30} < \sqrt{36}$, so $5 < \sqrt{30} < 6$, and $\sqrt{30} \approx 5.5$
- Estimate $(3.718)^2$.
  $3^2 < (3.7)^2 < 4^2$, so $9 < (3.7)^2 < 16$, and $(3.7)^2 \approx 14$

Know that:

- Sometimes there are different ways for finding an approximate answer.
- More than one approximation to the same calculation is possible.

For example:

- $467 \times 24 \approx 400 \times 25 = 10\,000$
  $467 \times 24 \approx 500 \times 20 = 10\,000$
- $677 \div 48 \approx 600 \div 40 = 15$
  $677 \div 48 \approx 700 \div 50 = 14$

Recognise what makes a 'good approximation'. Answer questions such as:

- Approximate: $\frac{127 \times 31}{19}$
- Explain how to estimate an answer to
  $(17.8 - 4.6) \div (11.4 + 9.7)$
  before you use a **calculator** to work it out.

Check the position of a decimal point in a multiplication by estimating the answer, e.g.

- $67.3 \times 98.02 \approx 70 \times 100 = 7000$
  and, using a **calculator**,
  $67.3 \times 98.02 = 6596.746$

Link to rounding (pages 42–5), and checking results of calculations (pages 110–11).

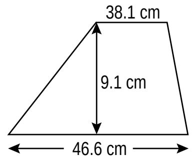
As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years. Understand the difference between and when to use $\approx$, $=$, $\neq$, $\equiv$.

Recognise when an exact answer is not needed and an estimate is sufficient, or when an exact answer is needed and an estimate is insufficient.

Find approximate areas by rounding lengths. For example, the approximate area of this trapezium is:

$\frac{1}{2} \times (40 + 50) \times 9\ \text{cm}^2$
$= \frac{1}{2} \times 810\ \text{cm}^2$
$= 405\ \text{cm}^2$



Know that 3, $^{22}/_7$ and 3.14 are all approximations to π. Use these values to calculate approximations to the areas and circumferences of circles.

Make and justify estimates of calculations such as:

- $(2095 \times 302) + 396$
- $3.75 \times (2.36 - 0.39)$
- $\frac{103 \times 0.44}{\sqrt{16.1}}$

for example, by rounding to one significant figure.

Recognise the effects of rounding up and down on a calculation. Discuss questions such as:

- Why is $6 \div 2$ a better approximation for $6.59 \div 2.47$ than $7 \div 2$?

Recognise when approximations to the nearest 10, 100 or 1000 are good enough, and when they are not.

Check the position of a decimal point in a multiplication by estimating the answer, e.g.

- $48.6 \times 0.078 \approx 50 \times 0.1 = 50 \div 10 = 5$
  and, using a **calculator**,
  $48.6 \times 0.078 = 3.7908$

Link to rounding (pages 42–5), checking results of calculations (pages 110–11), finding the area of a circle using an approximation to π (pages 236–7).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use efficient column methods for addition and subtraction of whole numbers, and extend to decimals

Continue to use and refine efficient methods for column addition and subtraction, while maintaining accuracy and understanding. Extend to decimals with up to two decimal places, including:

- sums and differences with different numbers of digits;
- totals of more than two numbers.

For example:

- 671.7 – 60.2
- 45.89 + 653.7
- 764.78 – 56.4
- 543.65 + 45.8
- 1040.6 – 89.09
- 76.56 + 312.2 + 5.07

See Y456 examples (pages 48–51).

Refine written methods of multiplication and division of whole numbers to ensure efficiency, and extend to decimals

Multiplication

Use written methods to support, record or explain multiplication of:
- a three-digit number by a two-digit number;
- a decimal with one or two decimal places by a single digit.

- 6.24 × 8 is approximately 6 × 8 = 48.

×	6	0.2	0.04	Answer
8	48	1.6	0.32	49.92

Progress from the 'grid' method (see Year 6) to using a standard procedure efficiently and accurately, with understanding.

- 673 × 24 is approximately 700 × 20 = 14 000.

```
                 673
             ×    24
673 × 20       13460
673 ×  4        2692
               16152
                1 1
```

- 6.24 × 8 is approximately 6 × 8 = 48 and is equivalent to 624 × 8 ÷ 100.

```
      624
  ×     8
     4992     4992 ÷ 100 = 49.92
      1 3
```

- 642.7 × 3 is approximately 600 × 3 = 1800 and is equivalent to 6427 × 3 ÷ 10.

```
     6427
  ×     3
    19281     19281 ÷ 10 = 1928.1
     1  2
```

See Y456 examples (pages 66–7).

Link to estimating calculations (pages 102–3, 110–11), and multiplying by powers of 10 (pages 38–9).

As outcomes, Year 8 pupils should, for example:

Consolidate the methods learned and used in previous years, and extend to harder examples of sums and differences with different numbers of digits.

For example:

- 44.8 + 172.9 + 87.36
- 5.05 + 3.9 + 8 + 0.97
- 14 – 3.98 – 2.9
- 32.7 + 57.3 – 45.17
- 18.97 + 2.9 – 17.36 – 28.4 + 5.04

Multiplication

Use written methods to multiply by decimals with up to two decimal places. Consider the approximate size of the answer in order to check the magnitude of the result. For example:

- 23.4 × 4.5 is approximately 23 × 5 = 115.

×	20	3	0.4	Check
4	80	12	1.6	93.6
0.5	10	1.5	0.2	+ 11.7
	90	13.5	1.8	105.3

Use a standard procedure to improve efficiency, maintaining accuracy and understanding.

- 1.89 × 23 is approximately 2 × 20 = 40, and is equivalent to 1.89 × 100 × 23 ÷ 100, or 189 × 23 ÷ 100.

```
               189
             ×  23
189 × 20      3780
189 ×  3       567
              4347
              1 1
```

Answer: 4347 ÷ 100 = 43.47

- 23.4 × 4.5 is approximately 23 × 5 = 115, and is equivalent to 23.4 × 10 × 4.5 × 10 ÷ 100, or 234 × 45 ÷ 100.

```
              234
            ×  45
234 × 40     9360
234 ×  5     1170
            10530
             1
```

Answer: 10 530 ÷ 100 = 105.3

Link to estimating calculations (pages 102–3, 110–11), and multiplying by powers of 10 (pages 38–9).

As outcomes, Year 9 pupils should, for example:

Use a standard column procedure for addition and subtraction of numbers of any size, including a mixture of large and small numbers with differing numbers of decimal places.

For example:

- 6543 + 590.005 + 0.0045
- 5678.98 – 45.7 – 0.6

Multiplication

Use a standard column procedure for multiplications equivalent to three digits by two digits. Understand where to put the decimal point for the answer. Consider the approximate size of the answer in order to check the magnitude of the result. For example:

- 64.2 × 0.43 ≈ 60 × 0.5 = 30, and is equivalent to 642 × 43 ÷ 1000.

```
     642
  ×   43
   25680
    1926
   27606
    1 1
```

Answer: 27606 ÷ 1000 = 27.606

Where appropriate, round the answer to a suitable number of decimal places *or significant figures. For example:*

- *0.0721 × 0.036 ≈ 0.07 × 0.04 = 0.0028, and is equivalent to 721 × 36 ÷ 10 000 000, or 0.002 595 6, or 0.0026 correct to 4 d.p.*
- *5.16 × 3.14 ≈ 5 × 3 = 15, and is equivalent to 516 × 314 ÷ 10 000, or 16.2024, or 16.2 correct to 3 s.f.*

Link to estimating calculations (pages 102–3, 110–11), and multiplying by powers of 10 (pages 38–9).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Refine written methods of multiplication and division of whole numbers to ensure efficiency, and extend to decimals with two places (continued)

Division

Use written methods to support, record or explain division of:

- a three-digit number by a two-digit number;
- a decimal with one or two decimal places by a single digit.

Progress from informal methods to using a standard algorithm efficiently and accurately, and with understanding.

For example:

- 3199 ÷ 7 is approximately 2800 ÷ 7 = 400.

```
 7)3199
 – 2800     7 × 400
    399
 –  350     7 ×  50
     49
 –   49     7 ×   7
      0
      Answer:  457
```

Refine methods to improve efficiency while maintaining accuracy and understanding.

- 109.6 ÷ 8 is approximately 110 ÷ 10 = 11.

```
  8)109.6                  8)109.6
 –    80.0   8 × 10        – 104.0   8 × 13
      29.6                     5.6
 –    24.0   8 ×  3            5.6   8 ×  0.7
       5.6                     0.0
 –     5.6   8 ×  0.7
       0.0
   Answer: 13.7
```

- 239.22 ÷ 6 is approximately 200 ÷ 5 = 40.

```
 6)239.22
 – 180.00   6 × 30
    59.22
 –  54.00   6 ×  9
     5.22
 –   4.80   6 ×  0.8
     0.42
 –   0.42   6 ×  0.07
     0.00
      Answer:  39.87
```

See Y456 examples (pages 68–9).

Link to estimating calculations (pages 102–3), and multiplying and dividing by powers of 10 (pages 38–9).

As outcomes, Year 8 pupils should, for example:

Division

Use a standard procedure for divisions equivalent to three digits by two digits, by transforming to an equivalent calculation with a non-decimal divisor.
Consider the approximate size of the answer in order to check the magnitude of the result.

For example:

- 91.8 ÷ 17 is approximately 100 ÷ 20 = 5.

```
 17)91.8
 –  85.0      17 × 5
     6.8
 –   6.8      17 × 0.4
     0.0
           Answer: 5.4
```

- 87.5 ÷ 16 is approximately 90 ÷ 15 = 6.

```
16)87.50
 – 80.00      16 × 5
    7.50
 –  6.40      16 × 0.4
    1.10
 –  0.96      16 × 0.06
    0.14
           Answer: 5.46 R 0.14
                   5.5 to 1 d.p.
```

- 428 ÷ 3.4 is approximately 400 ÷ 4 = 100 and is equivalent to 4280 ÷ 34.

```
 34)4280
 –  3400      34 × 100
     880
 –   680      34 ×  20
     200
 –   170      34 ×   5
      30.0
 –    27.2    34 ×   0.8
       2.80
 –     2.72   34 ×   0.08
       0.08

           Answer: 125.88 R 0.08
                   125.9 to 1 d.p.
```

Link to estimating calculations (pages 102–3), and multiplying and dividing by powers of 10 (pages 38–9).

As outcomes, Year 9 pupils should, for example:

Division

Use a standard procedure for divisions involving decimals by transforming to an equivalent calculation with a non-decimal divisor.
Consider the approximate size of the answer in order to check the magnitude of the result.

For example:

- 361.6 ÷ 0.8 is equivalent to 3616 ÷ 8.
- 547.4 ÷ 0.07 is equivalent to 54 740 ÷ 7.
- 0.048 ÷ 0.0035 is equivalent to 480 ÷ 35.
- 0.593 ÷ 6.3 is equivalent to 5.93 ÷ 63.

Where appropriate, round the answer to a suitable number of decimal places *or significant figures.*
For example:

- *0.0821 ÷ 0.78 ≈ 0.08 ÷ 0.8 = 0.1*
  and is equivalent to
  (821 ÷ 10 000) ÷ (78 ÷ 100)
  or
  (821 ÷ 78) ÷ 100
  or
  0.105 correct to 3 s.f.

Link to estimating calculations (pages 102–3), and multiplying and dividing by powers of 10 (pages 38–9)

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Carry out more complex calculations using the facilities on a calculator

Use, read and write, spelling correctly:
calculator, display, key, enter, clear, memory...

Know how to:

- Key in money calculations, and measurements of time, e.g. 4 hours 15 minutes is keyed in as 4.25 hours.
- Input a negative number.
- Use the bracket keys and select the correct key sequence to carry out calculations involving more than one step, e.g. to calculate 364 ÷ (23 + 17).
- Find whole-number remainders after division.
- Convert units of time, e.g. 1000 minutes to hours and minutes.
- Use the square and square root keys.
- Consider the approximate size of an answer before and after a calculation and, where necessary, check it appropriately, e.g. by performing the inverse operation.

For example:

- Use a **calculator** to work out:

  a. 7.6 – (3.05 – 1.7) b. $\frac{8.4 - 3.7}{8.4 + 3.7}$

Interpret the display on a calculator in different contexts (fractions, decimals, money, metric measures, time)

Know how to:

- Recognise a negative number in the display.
- Recognise how brackets are displayed.
- Interpret the display in the context of a problem, e.g. 109.2 may mean £109.20 in the context of money, 109 metres and 20 centimetres in the context of length, and 109 minutes and 12 seconds in the context of time.
- Read the display of, say, 91.333 333 3 after dividing 822 by 9 as '91 point three recurring', and know that 0.333 333 3 represents one third.
- Interpret a rounding error, e.g. when calculating 2 ÷ 7 × 7 some calculators may display 1.999 999 instead of 2.

For example:

- Convert 950 hours to days and hours.
  The display after dividing 950 by 24 will be 39.583 333. Subtract 39 from the answer to give the fraction of a day, then multiply by 24 to convert the fraction of a day back to hours.

See Y456 examples (pages 70–1).

Link to rounding numbers to one decimal place (pages 42–5).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
sign change key...

Know how to:
- Use the sign change or +/- key where appropriate.
- Use the memory and/or bracket keys, and select the correct key sequence to carry out complex calculations.
- Key in fractions, recognise the equivalent decimal form, and use this to compare and order fractions.
- Use the fraction key, including to enter time, e.g. 3 hours 25 minutes = $3\frac{25}{60}$ hours.
- Use the cube and cube root keys, if available.
- Consider the approximate size of an answer before and after a calculation and, where necessary, check it appropriately.

Use a **calculator** to evaluate correctly complex expressions such as those with brackets or where the memory function could be used.

For example:

- Use a calculator to work out

  $4 \times (6.78)^2$

Know how to:
- Recognise recurring decimals when they are rounded on the calculator, e.g. 2 ÷ 3 is displayed as 0.666 666 67.
- Recognise that if, for example, $\sqrt{3}$ is shown to be 1.732 051 then $(1.732\,051)^2 \approx 3$.

Link to rounding numbers to one or two decimal places (pages 42–5), converting fractions to decimals (pages 64–5), working with integers, powers and roots (pages 48–59).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
constant... reciprocal...

Know how to:
- Use the constant, π, sign change, power (x^y), root and fraction keys to evaluate expressions.
- *Use the reciprocal key ($\frac{1}{x}$).*

For example:
- Add on 101 repeatedly using the constant key. How long is the digit pattern maintained? Explain why.
- Find the circumference of a circle with radius 8 cm to two decimal places.
- Calculate 6^7, $\sqrt[4]{625}$, $\sqrt{(57.6/\pi)}$, $\sqrt{(15.5^2 - 3.7^2)}$.
- Use a calculator to work out the answer as a fraction for $\frac{12}{19} + \frac{17}{22}$.

Use a **calculator** to evaluate more complex expressions such as those with nested brackets or where the memory function could be used.

For example:

- Use a calculator to work out:

  a. $\dfrac{45.65 \times 76.8}{1.05 \times (6.4 - 3.8)}$

  b. $4.6 + (5.7 - (11.6 \times 9.1))$

  c. $\{(4.5)^2 + (7.5 - 0.46)\}^2$

  d. $\dfrac{5 \times \sqrt{(4.5^2 + 6^2)}}{3}$

*Understand how a **scientific calculator** presents large and small numbers in standard form, linking to work in science.*

Link to multiplying by powers of 10 and writing numbers in standard form (page 39).

*Use a **calculator** to investigate sequences involving a reciprocal function, such as:*

$$x \rightarrow \frac{1}{x-1}$$

Link to reciprocals (pages 82–3).

Link to rounding numbers to one or two decimal places (pages 42–5), converting fractions to decimals (pages 64–5), working with integers, powers and roots (pages 48–59).

Pupils should be taught to:

Use checking procedures, including working the problem backwards and considering whether the result is the right order of magnitude

As outcomes, Year 7 pupils should, for example:

Use the context of a problem to check whether an answer is sensible. For example:

- Check that the sum of two odd numbers, positive or negative, is an even number.
- When multiplying two large numbers together, check the last digit, e.g. 239 × 46 must end in a '4' because 6 × 9 = 54.
- Having multiplied a number by, for example, 3, the sum of the digits should be divisible by 3.

Discuss questions such as:

- A girl worked out the cost of 8 bags of apples at 47p a bag. Her answer was £4.06.
  Without working out the answer, say whether you think it is right or wrong.
- A boy worked out how many 19p stamps you can buy for £5. His answer was 25.
  Do you think he was right or wrong? Why?
- I buy six items costing 76p, 89p, 36p, £1.03, 49p and 97p.
  I give the shop assistant a £10 note and get £3.46 change.
  I immediately think the change is wrong. Without calculating the sum, explain why you think I am right.
- A boy worked out £2.38 + 76p on a calculator.
  The display showed 78.38.
  Why did the calculator give the wrong answer?

Use rounding to approximate and judge whether the answer is the right order of magnitude. For example:

• 2605 – 1897	is about	3000 – 2000
• 245 × 19	is about	250 × 20
• 786 ÷ 38	is about	800 ÷ 40
• 12% of 192	is about	10% of 200
• 1.74 × 16	lies between	1 × 16 = 16 and 2 × 16 = 32

Check by doing the inverse operation.
For example, use a **calculator** to check:

• 43.2 × 26.5 = 1144.8	with	1144.8 ÷ 43.2
• ³⁄₅ of 320 = 192	with	192 × 5 ÷ 3
• 3 ÷ 7 = 0.428 571 4…	with	7 × 0.428 571 4

Check by doing an equivalent calculation.
For example, check:

• 592 × 9 = 5328	with	(600 – 8) × 9 = 5400 – 72
	or	592 × (10 – 1) = 5920 – 592
• 44 × 99 = 4356	with	44 × (100 – 1) = 4400 – 44
	or	(40 + 4) × 99 = 3960 + 396

See Y456 examples (pages 72–3).

Link to making estimates and approximations of calculations (pages 102–3).

As outcomes, Year 8 pupils should, for example:

Use the context of a problem to check whether an answer is sensible. For example:

- When calculating a mean, check that it is within the range of the data. For example, the mean of 34, 21, 65, 89, 43, 29, 76, 79 must lie between 21 and 89.
- When using measurements, check the magnitude of the answer in the context of the problem.

Discuss questions such as:

- Will the answer to 75 ÷ 0.9 be smaller or larger than 75?
- A class of pupils was asked whether they preferred pop or classical music. They said:

Prefer classical	21%
Prefer pop	67%
Don't know	13%

  All results are correct to the nearest per cent but the three percentages add to 101%.
  Is this possible?
- Without using a calculator, pick out a possible answer to the calculation. Explain your choice.
  a. 47 × 59
     3443 or 2773 or 2887
  b. 456 × 0.48
     218.9 or 636 or 322.7

Use rounding to approximate and judge whether the answer is the right order of magnitude. For example:

- $\sqrt{7}$ lies between $\sqrt{4}$ and $\sqrt{9}$
  i.e. between 2 and 3
- Round to the nearest ten, e.g.
  $62/9$ is approximately $60/10 = 6$.
- Round to 'nice' numbers, e.g.
  $62/9$ is approximately $63/9 = 7$.

Check by doing the inverse operation.
For example, use a **calculator** to check:

- $\sqrt{7} = 2.645\,75\ldots$ with $(2.645\,75)^2$

Link to making estimates and approximations of calculations (pages 102–3), and checking the solution of an equation by substitution (pages 122–5).

As outcomes, Year 9 pupils should, for example:

Use the context of a problem to check whether an answer is sensible.

Discuss questions such as:

- The price of an audio system is reduced by 10%. Two months later the price increases by 10%. It does not return to its original price. Is this possible?
- Without using a calculator, pick out a possible answer to the calculation from the three possible answers given.
  a. $(398)^2$
     158 404 or 6344 or 161 484
  b. 365 ÷ 0.43
     849 or 84.9 or 157
  c. 67 ÷ 0.083
     87.2 or 8.72 or 807.2
  Explain your choice in each case.
- *Can a square have an exact area of 32 cm²? What about a circle?*

Link to making estimates and approximations of calculations (pages 102–3), checking the solution of an equation by substitution (pages 122–5), and checking that the sum of probabilities for all outcomes is 1 (pages 278–9).