Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Understand and use decimal notation and place value; multiply and divide integers and decimals by powers of 10

Use, read and write, spelling correctly:
place value, zero place holder, tenth, hundredth, thousandth... equivalent, equivalence...

Understand and use decimal notation and place value. Read and write any number from 0.001 to 1 000 000, knowing what each digit represents. For example, know that:

- In 5.239 the digit 9 represents nine thousandths, which is written as 0.009.
- The number 5.239 in words is 'five point two three nine', *not* 'five point two hundred and thirty-nine'.
- The fraction $5\frac{239}{1000}$ is read as 'five and two hundred and thirty-nine thousandths'.

Know the significance of 0 in 0.35, 3.05, 3.50, and so on.

Know that decimals used in context may be spoken in different ways. For example:

- 1.56 is spoken in mathematics as 'one point five six'.
- £1.56 is spoken as 'one pound fifty-six'.
- £1.06 is spoken as 'one pound and six pence'.
- £0.50 is spoken as 'fifty pence'.
- 1.56 km is sometimes spoken as 'one kilometre, five hundred and sixty metres'.
- 3.5 hours can be spoken as 'three and a half hours' or 'three hours and thirty minutes'.

Answer questions such as:

- Write in figures: four hundred and three thousand, and seventeen.
- Write in words: 4.236, 0.5, 35.08, …
- Write as a decimal the fraction six, and two hundred and forty-three thousandths.
- Make the largest and smallest number you can using:
  the digits 2, 0, 3, 4;
  the digits 2, 0, 3, 4, and a decimal point.

Add or subtract 0.1 and 0.01 to or from any number.

Count forwards or backwards from any number. For example:

- Count on in 0.1s from 4.5.
- Count back from 23.5 in 0.1s.
- Count on in 0.01s from 4.05.

Answer questions such as:

- What is 0.1 less than 2.0? What is 0.01 more than 2.09?
- What needs to be added or subtracted to change:
  27.48 to 37.48, 27.48 to 27.38, 27.48 to 26.38?
  5.032 to 5.037, 5.032 to 5.302?

See Y456 examples (pages 2–5, 28–9).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
billion, power, index…

Read and write positive integer powers of 10.

Know that:

1 hundred is $10 \times 10 = 10^2$
1 thousand is $10 \times 10 \times 10 = 10^3$
10 thousand is $10 \times 10 \times 10 \times 10 = 10^4$, etc.
1 million is 10^6
1 billion is 10^9, one thousand millions
(In the past, 1 billion was 10^{12}, one million millions, in the UK.)

Recognise that successive powers of 10 (i.e. 10, 10^2, 10^3, …) underpin decimal (base 10) notation.

Read numbers in standard form, e.g. read 7.2×10^3 as 'seven point two times ten to the power three'.

Link to using index notation (pages 56–9).

Add or subtract 0.001 to or from any number.

Answer questions such as:

- What is 0.001 more than 3.009?
  What is 0.001 more than 3.299?
  What is 0.002 less than 5?
  What is 0.005 less than 10?

- What needs to be added or subtracted to change:
  4.257 to 4.277? 6.132 to 6.139?
  5.084 to 5.053? 4.378 to 4.111?

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
standard (index) form… exponent…

Extend knowledge of integer powers of 10.

Know that:

$10^0 = 1$ $10^{-1} = 1/10^1 = \frac{1}{10}$
$10^1 = 10$ $10^{-2} = 1/10^2 = \frac{1}{100}$

Know the prefixes associated with powers of 10. Relate to commonly used units. For example:

10^9	giga	10^{-2}	centi
10^6	mega	10^{-3}	milli
10^3	kilo	10^{-6}	micro
		10^{-9}	nano
		10^{-12}	pico

Know the term *standard (index) form* and read numbers such as 7.2×10^{-3}.

Link to using index notation (pages 56–9) and writing numbers in standard form (pages 38–9).

Know that commonly used units in science, other subjects and everyday life are:

kilogram (kg) — SI unit	**metre (m)** — SI unit	
gram (g)	kilometre (km)	litre (l)
milligram (mg)	millimetre (mm)	millilitre (ml)

Pupils should be taught to:

Understand and use decimal notation and place value; multiply and divide integers and decimals by powers of 10 (continued)

As outcomes, Year 7 pupils should, for example:

Multiply and divide numbers by 10, 100 and 1000.

Investigate, describe the effects of, and explain multiplying and dividing a number by 10, 100, 1000, e.g. using a place value board, **calculator** or **spreadsheet**.

In particular, recognise that:

- Multiplying a positive number by 10, 100, 1000… has the effect of increasing the value of that number.
- Dividing a positive number by 10, 100, 1000… has the effect of decreasing the value of that number.
- When a number is multiplied by 10, the digits move one place to the left:

	3	4	.	1	2	34.12 multiplied by 10 = 341.2
3	4	1	.	2		

- When a number is divided by 10, the digits move one place to the right:

3	4	.	1		34.1 divided by 10 = 3.41
	3	.	4	1	

Complete statements such as:

4 × 10 = □	4 × □ = 400
4 ÷ 10 = □	4 ÷ □ = 0.04
0.4 × 10 = □	0.4 × □ = 400
0.4 ÷ 10 = □	0.4 ÷ □ = 0.004
□ ÷ 100 = 0.04	□ ÷ 10 = 40
□ × 1000 = 40 000	□ × 10 = 400

See Y456 examples (pages 6–7).

Link to converting mm to cm and m, cm to m, m to km… (pages 228–9).

As outcomes, Year 8 pupils should, for example:

Multiply and divide numbers by 0.1 and 0.01.

Investigate, describe the effects of, and explain multiplying and dividing a number by 0.1 and 0.01, e.g. using a **calculator** or **spreadsheet**.

In particular, recognise how numbers are increased or decreased by these operations.

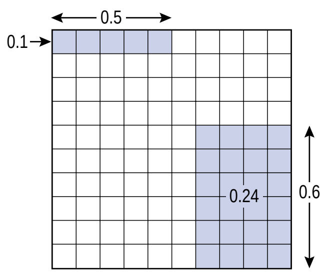
0.1 is equivalent to $\frac{1}{10}$ and 0.01 is equivalent to $\frac{1}{100}$, so:

- Multiplying by 0.1 has the same effect as multiplying by $\frac{1}{10}$ or dividing by 10. For example, 3×0.1 has the same value as $3 \times \frac{1}{10}$, which has the same value as $3 \div 10 = 0.3$, and 0.3×0.1 has the same value as $\frac{3}{10} \times \frac{1}{10} = \frac{3}{100} = 0.03$.
- Multiplying by 0.01 has the same effect as multiplying by $\frac{1}{100}$ or dividing by 100. For example, 3×0.01 has the same value as $3 \times \frac{1}{100}$, which has the same value as $3 \div 100 = 0.03$, and 0.3×0.01 has the same value as $\frac{3}{10} \times \frac{1}{100} = \frac{3}{1000} = 0.003$.
- Dividing by 0.1 has the same effect as dividing by $\frac{1}{10}$ or multiplying by 10. For example, $3 \div 0.1$ has the same value as $3 \div \frac{1}{10}$.
  (How many tenths in three? $3 \times 10 = 30$)
  $0.3 \div 0.1$ has the same value as $\frac{3}{10} \div \frac{1}{10}$.
  (How many tenths in three tenths? $0.3 \times 10 = 3$)
- Dividing by 0.01 has the same effect as dividing by $\frac{1}{100}$ or multiplying by 100. For example, $3 \div 0.01$ has the same value as $3 \div \frac{1}{100}$.
  (How many hundredths in three? $3 \times 100 = 300$)
  $0.3 \div 0.01$ has the same value as $\frac{3}{10} \div \frac{1}{100}$.
  (How many hundredths in three tenths? $0.3 \times 100 = 30$)

Complete statements such as:

$0.5 \times 0.1 = \square$	$0.8 \times \square = 0.08$
$0.7 \div 0.1 = \square$	$0.6 \div \square = 6$

Understand a diagrammatic explanation to show, for example, that $0.1 \times 0.5 = 0.05$, or $0.24 \div 0.6 = 0.4$.



Discuss the effects of multiplying and dividing by a number less than 1.

- Does division always make a number smaller?
- Does multiplication always make a number larger?

As outcomes, Year 9 pupils should, for example:

Multiply and divide by any integer power of 10.

For example:

- Calculate:

7.34×100	$37.4 \div 100$
46×1000	$3.7 \div 1000$
$8042 \times 10\,000$	$4982 \div 10\,000$
9.3×0.1	$0.27 \div 0.1$
0.63×0.01	$5.96 \div 0.01$

Link to converting mm² to cm², cm² to m², mm³ to cm³ and cm³ to m³ (pages 228–9).

Begin to write numbers in standard form, *expressing them as*

$A \times 10^n$ where $1 \le A < 10$, and n is an integer.

For example:

$734.6 = 7.346 \times 10^2$
$0.0063 = 6.3 \times 10^{-3}$

Know how to use the 'EXP' key on a ***calculator*** *to convert from index form.*

Answer questions such as:

- *Complete these. The first is done for you.*
  $3 \times 10^n = 300 \times 10^{n-2}$
  $0.3 \times 10^n = 30\,000 \times \square$
  $0.3 \times 10^n = 0.0003 \times \square$
  $3 \div 10^n = 0.003 \times \square$
  $0.3 \div 10^n = 300 \times \square$
  $0.003 \div 10^n = 3 \times \square$
- *Put these numbers in ascending order:*
  2×10^{-2}, 3×10^{-1}, 2.5×10^{-3}, 2.9×10^{-2}, 3.2×10^{-1}
- *Write these numbers in standard form:*
  a. *The population of the UK is 57 million.*
  b. *The dwarf pigmy goby fish weighs 0.000 14 oz.*
  c. *The shortest millipede in the world measures 0.082 inches.*
  d. *After the Sun, the nearest star is 24 800 000 000 000 miles away.*
- *The probability of dying before the age of 40 is 1 in 850, or 0.00118, or 1.8×10^{-3}.*

  These are the risks of dying from particular causes:

smoking 10 cigarettes a day	*1 in 200*
road accident	*1 in 8000*
accident at home	*1 in 260 000*
railway accident	*1 in 500 000*

  Write each of these as a probability in standard form.

Link to writing numbers in standard form in science and geography.

Pupils should be taught to:

Compare and order decimals

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
decimal number, decimal fraction, less than, greater than, between, order, compare, digit, most/least significant digit...
and use accurately these symbols: =, ≠, <, >, ≤, ≥.

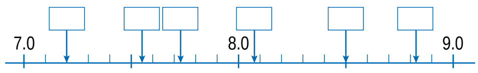
Know that to **order decimals**, digits in the same position must be compared, working from the left, beginning with the first non-zero digit. In these examples, the order is determined by:

• 0.325 < 0.345	the second decimal place;
• 3.18 km > 3.172 km	the second decimal place;
• 0.42 < 0.54	the first decimal place;
• 5.4 < 5.6 < 5.65	the first decimal place initially, then the second decimal place.

Know that when comparing measures it is necessary to convert all measures into the same units. For example:

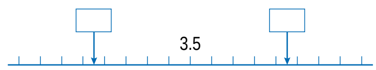
- Order these measurements, starting with the smallest:
  5 kg, 500 g, 0.55 kg
  45 cm, 1.23 m, 0.96 m
  £3.67, £3.71, 39p

Identify and estimate decimal fractions on a number line and find a number between two others by looking at the next decimal place. For example:



- Find the number that is half way between:
  3 and 4, 0.3 and 0.4, $^{-}3$ and 4, $^{-}4$ and 3.

- 3.5 lies half way between two other numbers.
  What could they be?



- Use a **computer simulation** to zoom in and out of a number line to compare and order decimals to at least 2 d.p.

- Use a **graphical calculator** to generate ten random numbers lying between 0 and 1, with a maximum of 2 d.p. Arrange the numbers in order. For example, enter:

  [Int] [(] [Ran#] [×] [1] [0] [0] [)] [÷] [1] [0] [0]

  then keep pressing the [EXE] button.

Use accurately the symbols <, >, ≤, ≥. For example:

- Place > or < between these:
  12.45 ☐ 12.54 $^{-}6$ °C ☐ $^{-}7$ °C
  3.424 ☐ 3.42 6.75 litres ☐ 675 millilitres

- Given that 31.6 ≤ *x* ≤ 31.8, give possible values for *x*:
  a. if *x* has 1 d.p. b. if *x* has two decimal places ...

See Y456 examples (pages 28–9).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
ascending, descending…

Know that to **order decimals**, digits in the same position must be compared, working from the left, beginning with the first non-zero digit.
In these examples, the order is determined by:

- 0.024 37 < 0.024 52 — the fourth decimal place;
- 3.1895 > 3.1825 — the third decimal place;
- 23.451 < 23.54 — the first decimal place;
- 5.465 < 5.614 < 5.65 — the first decimal place initially, then the second.

Extend to negative numbers, e.g. $^{-}0.0237 > {}^{-}0.0241$.

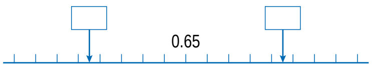
Know how to order data collected by measuring, and group the data in intervals. For example:

- When collecting data on pupils' heights, construct a frequency table using groups such as:
  $1.50 \le h < 1.55$ $\quad$ $1.55 \le h < 1.60$, etc.

Identify decimal fractions on a number line and find a number between two others by looking at the next decimal place. For example:

- Find the number that is half way between:
  0.03 and 0.04 $\quad$ $^{-}0.3$ and 0.4.

- 0.65 lies half way between two other numbers. What could they be?



- Use a **computer simulation** to zoom in and out of a number line to compare decimals to at least 3 d.p.

Link to graphs (pages 164–77), e.g. using a graphical calculator to zoom in on a graph as the axes change.

Use accurately the symbols <, >, ≤, ≥. For example:

- Place > or < between:
  0.503 ☐ 0.53 $\quad$ 3.2 metres ☐ 320 millimetres

- Given that $31.62 \le z \le 31.83$, discuss possible values for *z*. Understand that there is an infinite number of possible solutions.

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

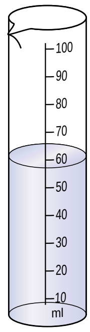
Round numbers, including to a given number of decimal places

As outcomes, Year 7 pupils should, for example:

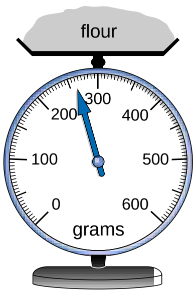
Use, read and write, spelling correctly:
round, nearest, to one decimal place (1 d.p.)... approximately...

Round positive whole numbers to the nearest 10, 100 or 1000, in mathematics and subjects such as science, design and technology, geography... For example:

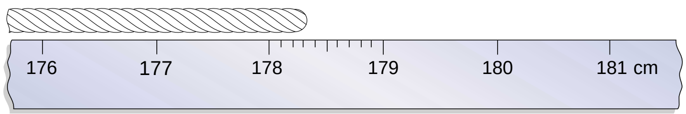
- What is the volume of the liquid in the measuring cylinder to the nearest 10 ml?



- What is the mass of the flour to the nearest 100 g?
  Estimate the mass of the flour to the nearest 10 g.



- How long is the rope to:
  a. the nearest 10 cm?
  b. the nearest 100 cm?
  c. the nearest cm?
  d. the nearest mm?



- How many people visited the Dome to the nearest 100?
  Was the headline correct?


The Daily Record 10 Feb

Nearly 20 thousand people visit Dome

15,437 people visited the Dome yesterday.

In other subjects, round whole numbers to the nearest 10, 100 or 1000 in order to classify them or put them in order. For example:

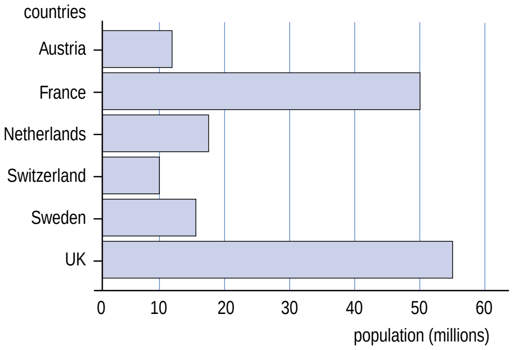
- In geography, round and then place in order:
  populations of towns, heights of mountains, weather data...
- In science, round and then place in order:
  the proportion of lead in the air at different places, the diameters of the planets...
- In design and technology, round and then place in order:
  the grams of fat in different foods...

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
recurring decimal…

Round positive whole numbers to a given power of 10, in mathematics and in other subjects.
For example:

- This chart shows the estimated population of six countries. Write in figures the approximate population of each country.



- On the chart above, Sweden is recorded as having an estimated population of 15 million. What is the highest/lowest population that it could actually have?

- There are 1 264 317 people out of work.
  Politician A says: 'We have just over 1 million people out of work.'
  Politician B says: 'We have nearly one and a half million people out of work.'
  Who is more accurate, and why?

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
significant figures, upper and lower bounds…
Read and write the 'approximately equal to' sign (≈).

Use rounding to make estimates.

For example:

- The population of the world is about 5300 million.

  The approximate populations of the four largest cities are:

Mexico City	21.5 million
Sao Paulo	19.9 million
Tokyo	19.5 million
New York	15.7 million

  The tenth largest city is Rio de Janeiro with a population of 11.9 million.

  Estimate the percentage of the world's population which lives in the ten largest cities.

- *A heavy metal in water kills fish when it reaches levels of more than 4 parts per million.*
  A lake contains 4.7 megalitres of water.
  How much heavy metal can be in the water for the fish to be safe, if 1 litre of the heavy metal has a mass of 2.4 kg?

Pupils should be taught to:

Round numbers, including to a given number of decimal places (continued)

As outcomes, Year 7 pupils should, for example:

Round positive whole numbers and decimals.

Know that if a measurement is half way between two numbers it is normally rounded up to the next number. Recognise that in some practical situations, such as a division problem, this may not be appropriate. For example:

- 124 children want to go on a school trip. If each coach holds 49 people, how many coaches are needed?
- I have 52 drawing pins. If each poster for my bedroom needs 6 pins, how many posters can I put up?
- A pupil in technology needs to cut a 1 metre length of wood into three pieces. How long should each piece be?

Link to understanding division (pages 82–5).

Round decimals to the nearest whole number or to one decimal place.

When rounding a decimal to a whole number, know that:
- if there are 5 or more tenths, then the number is rounded up to the next whole number; otherwise, the whole number is left unchanged;
- decimals with more than one decimal place are not first rounded to one decimal place, e.g. 7.48 rounds to 7, not to 7.5 which then rounds to 8.

When rounding a decimal such as 3.96 to one decimal place, know that the answer is 4.0, not 4, because the zero in the first decimal place is significant.

For example:
- 4.48 rounded to the nearest whole number is 4.
- 4.58 rounded to the nearest whole number is 5, and rounded to one decimal place is 4.6.
- 4.97 rounded to the nearest whole number is 5.
- 4.97 rounded to one decimal place is 5.0.

Answer questions such as:

- Round 5.28:
  a. to the nearest whole number;
  b. to one decimal place.

- Here are the winning heights and distances for some women's field events in an international competition. Round each height or distance:
  a. to the nearest whole number;
  b. to one decimal place.

Women's events	
High jump	2.09 metres
Long jump	7.48 metres
Shot-put	21.95 metres
Discus throw	76.80 metres
Javelin throw	80.00 metres

See Y456 examples (pages 12–13, 30–1, 56–7).

As outcomes, Year 8 pupils should, for example:

Recognise recurring decimals.

Recurring decimals contain an infinitely repeating block of one or more decimal digits.
For example:

- $\frac{1}{6}$ = 0.16666… is written as $0.1\dot{6}$
- $\frac{2}{11}$ = 0.181818… is written as $0.\dot{1}\dot{8}$

Fractions with denominators containing prime factors other than 2 or 5 will recur if written in decimal form.

Round decimals to the nearest whole number or to one or two decimal places.

For example, know that:

- 3.7452 rounded to the nearest whole number is 4, to one decimal place is 3.7, and to two decimal places is 3.75.
- 2.199 rounded to the nearest whole number is 2, to one decimal place is 2.2, and to two decimal places is 2.20.
- 6.998 rounded to two decimal places is 7.00.

When substituting numbers into expressions and formulae, know that rounding should not be done until the final answer has been computed.

Answer questions such as:

- Round 12.3599 to one decimal place
- Use a **calculator** to do these calculations.
  Write the answers to two decimal places.
  2 ÷ 3   3 ÷ 16   11 ÷ 9   9 ÷ 11   14 ÷ 17

Round decimals in context, selecting an appropriate number of decimal places to use when, for example:

- using decimal measurements for work on perimeter, area and volume;
- collecting measurements to use as data for statistics;
- calculating summary statistics, such as the mean;
- investigating recurring decimals;
- dividing;
- carrying out science experiments;
- measuring in design and technology or geography…

As outcomes, Year 9 pupils should, for example:

Round decimals to the nearest whole number or to one, two *and three* decimal places.

For example, know that:

- 3.0599 rounded to the nearest whole number is 3, rounded to 1 d.p. is 3.1, to 2 d.p. is 3.06, *and to 3 d.p. is 3.060.*
- 9.953 rounded to the nearest whole number is 10, to 1 d.p. is 10.0, and to 2 d.p. is 9.95.
- $\frac{22}{7}$ is an approximation to π and can be given as 3.14 to 2 d.p. or 3.143 correct to 3 d.p.

Know that rounding should not be done until a final result has been computed.

Answer questions such as:

- Use a **calculator** to evaluate $\frac{1}{850}$ correct to one decimal place.

Round decimals in context. Select an appropriate number of decimal places to use, knowing at which stage to round when, for example:

- approximating π in circle measurements and calculations;
- making measurements in mathematics and other subjects;
- when presenting results of calculations in geometrical and statistical contexts;
- when substituting decimals into expressions and formulae.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Round numbers, including to a given number of decimal places or significant figures (continued)	

As outcomes, Year 8 pupils should, for example:	As outcomes, Year 9 pupils should, for example:

Understand upper and lower bounds. *For example:*

- *For* ***discrete data*** *such as:*

  The population p of Sweden to the nearest million is 15 million.

  know that the least population could be 14 500 000 and the greatest population could be 15 499 999; understand that this can be written as:

  $$14\,500\,000 \le p < 15\,500\,000$$

- *For* ***continuous data*** *such as measurements of distance:*

  The distance d km from Exeter to Plymouth is 62 km to the nearest km.

  know that the shortest possible distance is 61.5 km and the longest possible distance is 62.5 km, which can be written as:

  $$61.5 \le d < 62.5$$

Round numbers to a given number of significant figures. *Know, for example, that:*

- *5.78 is 5.8 to two significant figures (2 s.f.).*
- *34.743 is 35 to 2 s.f. and 34.7 to 3 s.f.*
- *5646 is 6000 to 1 s.f., 5600 to 2 s.f. and 5650 to 3 s.f.*
- *0.004 36 is 0.004 to 1 s.f. and 0.0044 to 2 s.f.*

Know when to insert zeros as place holders to indicate the degree of significance of the number. For example, 1.4007 is 1.40 to 3 s.f.

Use numbers to a given number of significant figures to work out an approximate answer. For example:

- *The area of a circle with radius 7 cm is approximately 3×50 cm².*
  Compare this answer with the approximations $\frac{22}{7} \times 7 \times 7$ cm² and $3.14 \times 7 \times 7$ cm², and with $\pi \times 7 \times 7$ cm² calculated using the π key on a calculator.

Give answers to calculations to an appropriate number of significant figures. For example:

- $\dfrac{65 + 78}{41 \times 56} \approx 0.0623$ *to 3 s.f.*

- $5.84 + \dfrac{3.26 + 4.17}{1.23} \approx 12$ *to 2 s.f.*

Pupils should be taught to:

Order, add, subtract, multiply and divide positive and negative numbers

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
integer, positive, negative, plus, minus…
and know that ¯6 is read as 'negative six'.

Order integers and position them on a number line. For example:

- Put a > or < sign between these pairs of temperatures:
  ¯6 °C ☐ 4 °C   6°C ☐ ¯4 °C   ¯6 °C ☐ ¯4 °C   ¯4 °C ☐ ¯6 °C
- On a number line, mark numbers half way between two given negative numbers, or between a given positive number and a given negative number.
- Use a **graphical calculator** to generate ten random numbers lying between ¯20 and ⁺20, then arrange them in order. For example, enter:

  [Int] [(] [Ran#] [×] [4] [0] [)] [−] [2] [0]

  then keep pressing the [EXE] button.

Link to plotting coordinates in all four quadrants (pages 218–19).

Begin to add and subtract integers.

Extend patterns such as:

2 + 1 = 3	¯3 – 1 = ¯4
2 + 0 = 2	¯3 – 0 = ¯3
2 + ¯1 = 1	¯3 – ¯1 = ¯2
2 + ¯2 = 0	¯3 – ¯2 = ¯1
2 + ¯3 = ¯1	¯3 – ¯3 = 0

Use negative number cards to help answer questions such as:

¯3 + ¯5 = ☐	¯13 + ¯25 = ☐
¯146 + ¯659 = ☐	¯99 + ¯99 = ☐
¯9 – ¯4 = ☐	¯43 – ¯21 = ☐
¯537 – ¯125 = ☐	¯99 – ¯99 = ☐

Answer open-ended questions such as:

- The answer to a question was ¯8.
  What was the question?
- The result of subtracting one integer from another is ¯2.
  What could the two integers be?
- The temperature is below freezing point.
  It falls by 10 degrees, then rises by 7 degrees.
  What could the temperature be now?

Solve simple puzzles or problems involving addition and subtraction of positive and negative numbers, such as:

- Complete this magic square.

¯5	2	¯6
	¯8	¯1

Link to substituting positive and negative numbers in expressions and formulae (pages 138–41).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

Order positive and negative decimals.

See ordering decimals (pages 40–1).

Add and subtract integers.

Understand that 1 add ${}^-1$ is zero, and use this to calculate, for example:

$1 + 1 + {}^-1 + 1 + {}^-1 = \square$ | ${}^-1 + 1 = \square$

$3 + {}^-3 = \square$ | ${}^-29 + 29 = \square$
$827 + {}^-827 = \square$ | ${}^-3 + \square = 0$

$62 + {}^-51 = \square$ | ${}^-87 + 90 = \square$
$40 + {}^-30 + 20 + {}^-10 = \square$ | ${}^-3 + \square = 7$

Recognise that:
- $0 - {}^-1$ has the same value as $(1 + {}^-1) - {}^-1 = 1$;
- $0 - 1$ has the same value as $(1 + {}^-1) - 1 = {}^-1$.

Use this to calculate, for example:
$5 - {}^-3 = \square$ ${}^-4 - {}^-5 = \square$ ${}^-2 - \square = 7$

Use the sign change key on a **calculator** to work out:

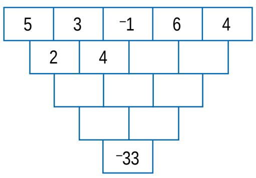
$134 - {}^-965$ ${}^-437 + 356$ $538 - {}^-702$
$468 + {}^-727$ ${}^-815 + {}^-768$ ${}^-646 - {}^-573$

Solve puzzles such as:

- Complete this table.

a	19	6	7	3	${}^-4$		${}^-8$
b	12	14	${}^-4$		${}^-6$	${}^-5$	
a – *b*				8		2	0

- In this ‘wall’, subtract the right-hand from the left-hand number to find the number directly below. Fill in the empty boxes.



Link to substituting positive and negative numbers in expressions and formulae (pages 138–41).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Order, add, subtract, multiply and divide positive and negative numbers (continued)

As outcomes, Year 7 pupils should, for example:

Use positive and negative numbers in context.

For example, find:

- the final position of an object after moves forwards and backwards along a line;
- a total bank balance after money is paid in and taken out;
- the total marks in a test of 10 questions, with $^{+}2$ marks for a correct answer and $^{-}1$ mark for an incorrect answer;
- the total of scores above and below par in a round of golf;
- the mean of a set of temperatures above and below zero…

Know how to, for example:

- find the distance between two floors using a lift, including above and below ground level;
- calculate game scores which include positive and negative points;
- identify measurements above and below sea-level, using contour lines on maps;
- interpret world weather charts to find differences in temperatures around the globe;
- identify the level of accuracy in measurements, e.g. 20 cm ± 0.5 cm…

Link to work in other subjects.

As outcomes, Year 8 pupils should, for example:

Multiply and divide positive and negative numbers.

Link known multiplication tables to negative number multiplication tables. For example:

- ${}^{-}2 \times 1 = {}^{-}2$, ${}^{-}2 \times 2 = {}^{-}4$, ${}^{-}2 \times 3 = {}^{-}6$
  and so on …
- Write tables, continuing the pattern:

$2 \times 2 = 4$	$2 \times {}^{-}2 = {}^{-}4$
$1 \times 2 = 2$	$1 \times {}^{-}2 = {}^{-}2$
$0 \times 2 = 0$	$0 \times {}^{-}2 = 0$
${}^{-}1 \times 2 = {}^{-}2$	${}^{-}1 \times {}^{-}2 = 2$
${}^{-}2 \times 2 = {}^{-}4$	${}^{-}2 \times {}^{-}2 = 4$
${}^{-}3 \times 2 = {}^{-}6$	${}^{-}3 \times {}^{-}2 = 6$

Complete a multiplication table. Shade positive and negative numbers, and zero, using different colours.

Look for patterns.

×	-3	-2	-1	0	1	2	3
3	-9	-6	-3	0	3	6	9
2	-6	-4	-2	0	2	4	6
1	-3	-2	-1	0	1	2	3
0	0	0	0	0	0	0	0
-1	3	2	1	0	-1	-2	-3
-2	6	4	2	0	-2	-4	-6
-3	9	6	3	0	-3	-6	-9

Recognise that division by a negative number is the inverse of multiplication by a negative number. Use this, and the negative number multiplication tables, to show, for example, that ${}^{-}4 \div {}^{-}2 = 2$, and relate this to the question 'How many ${}^{-}2$s in ${}^{-}4$?'

For a fact such as ${}^{-}3 \times 2 = {}^{-}6$, write three other facts, i.e. $2 \times {}^{-}3 = {}^{-}6$, ${}^{-}6 \div 2 = {}^{-}3$, ${}^{-}6 \div 3 = {}^{-}2$.

Answer questions such as:

- How many negative twos make negative four? (Two.)
- The answer to a question was ${}^{-}24$.
  What was the question?

Use the sign change key on a **calculator** to work out:

$48 \times {}^{-}53$	${}^{-}74 \times 3$	$9.02 \div {}^{-}22$
$68 \times {}^{-}49$	${}^{-}8 \times {}^{-}73.7$	${}^{-}6450 \div {}^{-}15$

Solve puzzles such as:

- Complete this multiplication grid. Find two ways to do it.

×		4	-9	
		-8	18	
-3		-12		
	35			-14
				12

Extend to the distributive law. For example:

$${}^{-}1 \times (3 + 4) = {}^{-}1 \times 7 = {}^{-}7$$
$${}^{-}1 \times (3 + 4) = ({}^{-}1 \times 3) + ({}^{-}1 \times 4) = {}^{-}3 + {}^{-}4 = {}^{-}7$$

Link to substituting positive and negative numbers in expressions and formulae (page 138–41).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Recognise and use multiples, factors and primes; use tests of divisibility

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
multiple, lowest common multiple (LCM), factor, common factor, highest common factor (HCF), divisor, divisible, divisibility, prime, prime factor, factorise…

Know that a prime number has two and only two distinct factors (and hence that 1 is not a prime number).

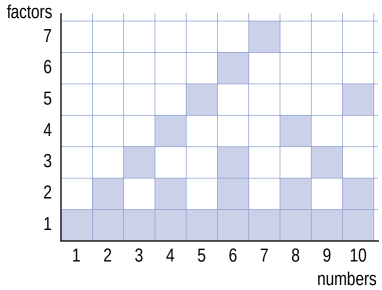
Know the prime numbers up to 30 and test whether two-digit numbers are prime by using simple tests of divisibility, such as:
- 2 the last digit is 0, 2, 4, 6 or 8;
- 3 the sum of the digits is divisible by 3;
- 4 the last two digits are divisible by 4;
- 5 the last digit is 0 or 5;
- 6 it is divisible by both 2 and 3;
- 8 half of it is divisible by 4;
- 9 the sum of the digits is divisible by 9.

Explore links between factors, primes and multiples.
For example:

- Find the primes to 100 by using the sieve of Eratosthenes. On a hundred square, colour in 1, then the multiples of 2 that are greater than 2 in one colour, the multiples of 3 that are greater than 3 in another colour… so that the remaining uncoloured numbers are the primes.

Find the factors of a number.

- Make a ‘factor finder’.



- Find the factors of a number by checking for divisibility by primes. For example, to find the factors of 123, check mentally or otherwise if the number divides by 2, then 3, 5, 7, 11…

- Find all the pairs of factors of non-prime numbers. For example:
  the pairs of factors of 51 are 1×51 and 3×17;
  the pairs of factors of 56 are 1×56, 2×28, 4×14, 7×8.

Use factors when appropriate to calculate mentally, as in:

$$35 \times 12 = 35 \times 2 \times 6 = 70 \times 6 = 420$$

$$\frac{144}{36} = \frac{12 \times 12}{3 \times 12} = \frac{12}{3} = 4$$

Link to cancelling fractions (pages 60–3).

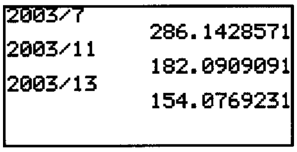
As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
prime factor decomposition...

Apply tests of divisibility for 12, 15, 18… by applying two simpler tests. For example, for:
- 15 the number is divisible by 3 and divisible by 5;
- 18 the number is even and divisible by 9.

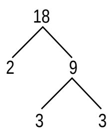
Use a **calculator** to explore divisibility. For example:

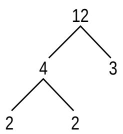
- Is 2003 a prime number?



Find the prime factor decomposition of a number.

Use factor trees to find prime factors and write non-prime numbers as the products of prime factors. For example, $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$.





$18 = 2 \times 3 \times 3$
$= 2 \times 3^2$

$12 = 2 \times 2 \times 3$
$= 2^2 \times 3$

Divide by prime numbers, in ascending order, to find all the prime factors of a non-prime number. Write the number as a product of prime factors.

2	24
2	12
2	6
3	3
	1

$24 = 2 \times 2 \times 2 \times 3$
$= 2^3 \times 3$

2	180
2	90
3	45
3	15
5	5
	1

$180 = 2 \times 2 \times 3 \times 3 \times 5$
$= 2^2 \times 3^2 \times 5$

Use factors when appropriate to calculate, as in:

$64 \times 75 = 64 \times 25 \times 3$
$= 1600 \times 3$
$= 4800$

$\sqrt{576} = \sqrt{(3 \times 3 \times 8 \times 8)}$
$= 3 \times 8$
$= 24$

Link to cancelling fractions (pages 60–3).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Recognise and use multiples, factors and primes; use tests of divisibility (continued)

As outcomes, Year 7 pupils should, for example:

Find the lowest common multiple (LCM) of two numbers, such as: 6 and 8; 25 and 30.

6 times table: 6 12 18 (24) 30…

8 times table: 8 16 (24) 32…

The lowest common multiple of 6 and 8 is 24.

Find the highest common factor (HCF) of two numbers, such as: 18 and 24; 40 and 65.

The factors of 18 are 1 2 3 (6) 9 18

The factors of 24 are 1 2 3 4 (6) 8 12 24

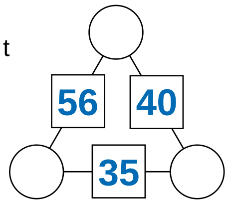
1, 2, 3 and 6 are common factors of 18 and 24, so 6 is the highest common factor of 18 and 24.

Link to cancelling fractions (pages 60–3).

See Y456 examples (pages 18–21).

Investigate problems such as:

- Which numbers less than 100 have exactly three factors?
- What number up to 100 has the most factors?
- Find some prime numbers which, when their digits are reversed, are also prime.
- There are 10 two-digit prime numbers that can be written as the sum of two square numbers. What are they?
- Write a number in each circle so that the number in each square is the product of the two numbers on either side of it.

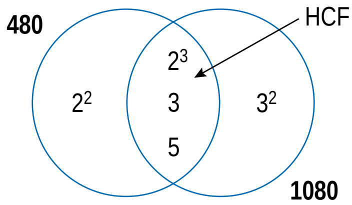


As outcomes, Year 8 pupils should, for example:

Use prime factors to find the highest common factor and lowest common multiple of a set of numbers.
For example:

Find the HCF and LCM of 480 and 1080.

480	$= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$
	$= 2^5 \times 3 \times 5$
1080	$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$
	$= 2^3 \times 3^3 \times 5$
HCF	$= 2^3 \times 3 \times 5$
LCM	$= 2^5 \times 3^3 \times 5$



Link to cancelling fractions (pages 60–3), and adding and subtracting fractions (pages 66–7).

Investigate problems such as:

- Show that 60 has exactly 12 factors.
  Find three more numbers less than 100 with exactly 12 factors.

- The sum of the digits of 715 is 13, and 13 is a factor of 715. What other three-digit numbers have this property?

As outcomes, Year 9 pupils should, for example:

Use prime factor decomposition to find the lowest common multiple of denominators of fractions in order to add or subtract them efficiently.
For example:

- $\frac{31}{56} + \frac{29}{70} = \frac{155 + 116}{280}$
  because $56 = 2^3 \times 7$ and $70 = 2 \times 5 \times 7$ so
  LCM $= 2^3 \times 5 \times 7 = 280$.

- $\frac{17}{28} - \frac{12}{38} = \frac{323 - 168}{532}$
  because $28 = 2^2 \times 7$ and $38 = 2 \times 19$ so
  LCM $= 2^2 \times 7 \times 19 = 532$.

Link to adding and subtracting fractions (pages 66–7).

Use prime factor decomposition to find the highest common factor in order to cancel fractions.

Link to cancelling fractions (pages 60–3), and multiplying and dividing fractions (pages 68–9).

Investigate problems such as:

- Take any two-digit number. Reverse the digits. Subtract the smaller number from the larger. Prove that the difference is always divisible by 9.
  Let the number be $10t + u$.
  Reversing the digits gives $10u + t$.
  The difference is
  $10t + u - 10u - t = 9t - 9u = 9(t - u)$
  showing that 9 is always a factor.

- Prove that a two-digit number in which the tens digit equals the units digit is always divisible by 11.
  The number is of the form $10t + t = 11t$.

- Prove that a three-digit number in which the sum of the hundreds digit and the units digit equals the tens digit is always divisible by 11.
  The number is of the form
  $100h + 10t + (t - h) = 99h + 11t = 11(9h + t)$

Find the common factors of algebraic expressions.
For example:

- $2x^2yz$ and $3wxy$ have a common factor xy.

- *Find the HCF and LCM of a^5b^4 and a^4b^4.*
  HCF = a^4b^4 LCM = a^5b^4

- *$(x - 1)(2x + 3)^3$ and $(x - 1)^2(2x - 3)$ have the common factor $(x - 1)$.*

Link to finding common factors in algebra (pages 116–17).

Pupils should be taught to:

Recognise squares and cubes, and the corresponding roots; use index notation and simple instances of the index laws

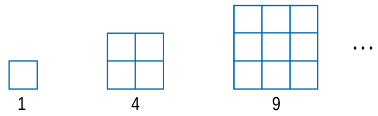
As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
property, consecutive, classify…
square number, squared, square root… triangular number…
the notation 6^2 as *six squared* and the square root sign √.

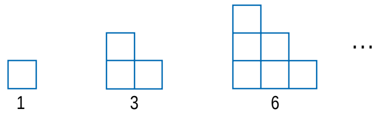
Use index notation to write squares such as 2^2, 3^2, 4^2, …

Recognise:

- squares of numbers 1 to 12 and the corresponding roots;

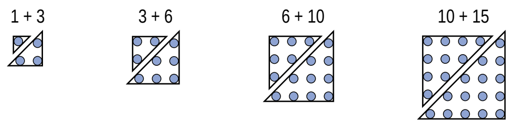


- triangular numbers: 1, 3, 6, 10, 15, …



Work out the values of squares such as 15^2, 21^2.

Investigate the relationship between square numbers and triangular numbers, using interlocking cubes or pegboard.



Link to generating sequences from practical contexts (pages 146–7).

Square roots

Find square roots of multiples of 100 and 10 000 by factorising. For example, find:

- $\sqrt{900} = \sqrt{(9 \times 100)}$
  $= \sqrt{9} \times \sqrt{100} = 3 \times 10 = 30$
- $\sqrt{160\,000} = \sqrt{(16 \times 100 \times 100)}$
  $= \sqrt{16} \times \sqrt{100} \times \sqrt{100} = 4 \times 10 \times 10 = 400$

Use a **calculator**, including the square root key, to find square roots, rounding as appropriate.

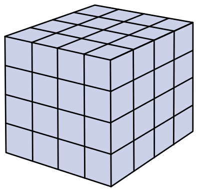
Recognise that squaring and finding the square root are the inverse of each other.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
cube number, cubed, cube root... power...
the notation 6^3 as *six cubed*, and 6^4 as *six to the power 4...* and the cube root sign $\sqrt[3]{}$.

Use index notation to write cubes and small positive integer powers of 10.

Know cubes of 1, 2, 3, 4, 5 and 10 and the corresponding roots.



$4^3 = 64$

Work out the values of cubes, such as 6^3, $^-9^3$, $(0.1)^3$.

Know that $100 = 10 \times 10 = 10^2$, and that successive powers of 10 (10, 10^2, 10^3, ...) underpin decimal (base 10) notation. For example:

1 thousand is 10^3;
10 thousand is 10^4;
1 million is 10^6;
1 billion is 10^9 (one thousand millions).

Link to place value (pages 36–7), prime factor decomposition of a number and tree diagrams (pages 52–3), and generating sequences from practical contexts (pages 146–7).

Squares, cubes and square roots

Know that a positive integer has two square roots, one positive and one negative; by convention the square root sign $\sqrt{}$ denotes the positive square root.

Find square roots by factorising, for example:
$\sqrt{196} = \sqrt{(4 \times 49)} = 2 \times 7 = 14$

Find an upper and lower bound for a square root by comparing with the roots of two consecutive square numbers:
$\sqrt{4} < \sqrt{7} < \sqrt{9}$ so $2 < \sqrt{7} < 3$

Use a **calculator** to find cubes, squares and estimate square roots, including using the square root key. For example:

- Find the square root of 12.

3^2 = 9 (3 to the power 2)
4^2 = 16
so the square root of 12 lies between 3 and 4.

```
3^2
        9
4^2
       16
3.5^2
    12.25
3.4^2
```

Try 3.5, and so on.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
index, indices, index notation, index law...

Use index notation for small integer powers.
For example:

- $19^3 = 6859$ $6^5 = 7776$ $14^4 = 38\,416$

Know that $x^0 = 1$, for all values of x.

Know that:
$10^{-1} = \frac{1}{10} = 0.1$ $10^{-2} = \frac{1}{100} = 0.01$

Know how to use the x^y key on a **calculator** to calculate powers.

Recognise applications of indices in biology, where cells and organisms grow by doubling, giving rise to the powers of 2.

Link to writing numbers in standard form (pages 38–9).

Square roots and cube roots

Know that:
- $\sqrt{a} + \sqrt{b} \neq \sqrt{(a + b)}$

Know that:
- there are two square roots of a positive integer, one positive and one negative, written as $\pm\sqrt{}$;
- the cube root of a positive number is positive and the cube root of a negative number is negative.

Use **ICT** to estimate square roots or cube roots to the required number of decimal places. For example:

- Estimate the solution of $x^2 = 70$.

  The positive value of x lies between 8 and 9, since $8^2 = 64$ and $9^2 = 81$.
  Try numbers from 8.1 to 8.9 to find a first approximation lying between 8.3 and 8.4.
  Next try numbers from 8.30 to 8.40.

Link to using trial and improvement and ICT to find approximate solutions to equations (pages 132–5).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Recognise squares and cubes, and the corresponding roots; use index notation and simple instances of the index laws (continued)	Investigate problems such as: • Without using a calculator, find a number that when multiplied by itself gives 2304. • Describe the pattern formed by the last digits of square numbers. Do any numbers not appear as the last digits? Could 413 be a square number? Or 517? • Can every square number up to 12×12 be expressed as the sum of two prime numbers? • Some triangular numbers are equal to the sum of two other triangular numbers. Find some examples. **See Y456 examples (pages 20–1).**

As outcomes, Year 8 pupils should, for example:

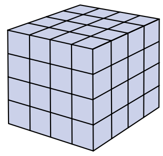
Investigate problems such as:

- Using a **calculator**, find two consecutive numbers with a product of 7482.
- Some numbers are equal to the sum of two squares: for example, $34 = 3^2 + 5^2$.
  Which numbers less than 100 are equal to the sum of two squares?
  Which can be expressed as the sum of two squares in at least two different ways?
- What are the 20 whole numbers up to 30 that can be written as the difference of two squares?
- Find the smallest number that can be expressed as the sum of two cubes in two different ways.
- What are the three smallest numbers that are both triangular and square?

As outcomes, Year 9 pupils should, for example:

Investigate problems such as:

- Estimate the cube root of 20.
- The outside of a cube made from smaller cubes is painted blue. How many small cubes have 0, 1, 2 or 3 faces painted blue? Investigate.



- *Three integers, each less than 100, fit the equation $a^2 + b^2 = c^2$.*
  What could the integers be?

Link to Pythagoras' theorem (pages 186–9); graphs of quadratic and cubic functions (pages 170–1).

Use simple instances of the index laws and start to multiply and divide numbers in index form.

Recognise that:

- indices are added when multiplying, e.g.
  $4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4)$
  $= 4 \times 4 \times 4 \times 4 \times 4$
  $= 4^5 = 4^{(3+2)}$
- indices are subtracted when dividing, e.g.
  $4^5 \div 4^2 = (4 \times 4 \times 4 \times 4 \times 4) \div (4 \times 4)$
  $= 4 \times 4 \times 4$
  $= 4^3 = 4^{(5-2)}$
- $4^2 \div 4^5 = 4^{(2-5)} = 4^{-3}$
- $7^5 \div 7^5 = 7^0 = 1$

Generalise to algebra. Apply simple instances of the index laws (small integral powers), as in:

- $n^2 \times n^3 = n^{2+3} = n^5$
- $p^3 \div p^2 = p^{3-2} = p$

Know and use the general forms of the index laws for multiplication and division of integer powers.

$p^a \times p^b = p^{a+b}$, $p^a \div p^b = p^{a-b}$, $(p^a)^b = p^{ab}$

Begin to extend understanding of index notation to negative and fractional powers; recognise that the index laws can be applied to these as well.

2^{-4}	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3	2^4
$\frac{1}{2^4} = \frac{1}{16}$	$\frac{1}{2^3} = \frac{1}{8}$	$\frac{1}{2^2} = \frac{1}{4}$	$\frac{1}{2^1} = \frac{1}{2}$	1	2	4	8	16

Know the notation $5^{1/2} = \sqrt{5}$ and $5^{1/3} = \sqrt[3]{5}$.

Extend to simple surds (unresolved roots):

- $\sqrt{3} \times \sqrt{3} = 3$
- $\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{3}$
- $\sqrt{32} = \sqrt{(2 \times 16)} = \sqrt{2} \times \sqrt{16} = 4\sqrt{2}$

- *Can a square have an exact area of 32 cm²?*
  If so, what is its exact perimeter?