Pupils should be taught to:

Use the vocabulary of probability

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
fair, unfair, likely, unlikely, equally likely, certain, uncertain, probable, possible, impossible, chance, good chance, poor chance, no chance, fifty-fifty chance, even chance, likelihood, probability, risk, doubt, random, outcome...

Use vocabulary and ideas of probability, drawing on experience. For example:

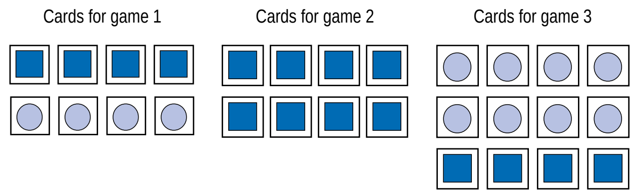
- Match one of these words to each statement below:

  CERTAIN **LIKELY** **UNLIKELY** **IMPOSSIBLE**

  a. I will eat a packet of crisps today.
  b. Next year, there will be 54 Fridays.
  c. I will leave the classroom through the door.
  d. The sun will rise tomorrow in the east.
  e. I will see David Beckham on my way home.

- Discuss the risk or chance of:
  a. injury in different sports;
  b. road accidents at different times of the day and year;
  c. dying before the age of 70 in different countries;
  d. a cyclone happening in England.

- A class is going to play three games.
  In each game some cards are put into a bag.
  Each card has a square or a circle on it.
  One card will be taken out, then put back.
  If it is a circle, the girls will get a point.
  If it is a square, the boys will get a point.



  a. Which game are the girls most likely to win? Why?
  b. Which game are the boys least likely to win? Why?
  c. Which game is impossible for the girls to win?
  d. Which game are the boys certain to win?
  e. Which game is it equally likely that the boys or girls win?
  f. Are any of the games unfair? Why?

- Use a line of large digit cards (1 to 10), face down and in random order.
  Turn cards over, one at a time.
  Indicate whether the next card turned is likely to be higher or lower than the card just turned.
  Give reasons for each response.

See Y456 examples (pages 112–13).

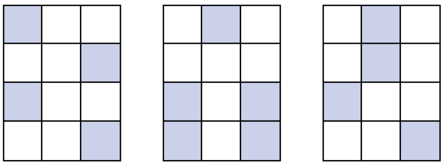
As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
event, theory, sample, sample space, biased…

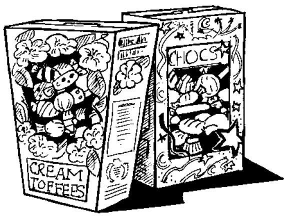
Use the vocabulary of probability when interpreting the results of an experiment; appreciate that random processes are unpredictable. For example:

- Think of an event where:
  a. the outcome is certain;
  b. the outcome is impossible;
  c. the outcome has an even chance of occurring.

- Three different scratch cards have some hidden shaded squares. You can scratch just one square, choosing at random.
  On which card are you most likely to reveal a shaded area? Why?



- Two boxes of sweets contain different numbers of hard- and soft-centred sweets.



Box 1 has 8 hard-centred sweets and 10 with soft centres.
Box 2 has 6 hard-centred sweets and 12 with soft centres.
Kate only likes hard-centred sweets. She can pick a sweet at random from either box. Which box should she pick from? Why?

Kate is given a third box of sweets with 5 hard-centred sweets and 6 with soft centres.
Which box should Kate choose from now? Why?

As outcomes, Year 9 pupils should, for example:

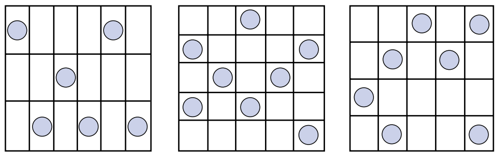
Use vocabulary from previous years and extend to:
exhaustive, independent, mutually exclusive, relative frequency, limit, tree diagram…
and the notation p(n) for the probability of event n.

Use the vocabulary of probability in interpreting results involving uncertainty and prediction.
For example:

- Discuss statements such as:
  a. As so many thousands of people play the Lottery each week somebody is **certain** to win the jackpot.
  b. Every morning I drop my toast and it has landed butter-side down for the last three mornings. It couldn't **possibly** happen again today.
  c. It can either rain or be fine, so tomorrow there is a **50% chance** of rain.
  d. To play a board game you must throw a six to start. Amin says: 'I'm **not** lucky, I'll **never** be able to start.'
  e. The risk of being killed in a road accident is about 1 in 8000, and of dying of a heart attack is 1 in 4.

- In a computer 'minefield' game, 'mines' are hidden on grids. When you land randomly on a square with a mine, you are out of the game.

  a. The circles indicate where the mines are hidden on three different grids.



Grid 1 Grid 2 Grid 3

  On which of the three grids is it hardest to survive?

  b. On which of these grids is it hardest to survive?
  X. 10 mines on an 8 by 8 grid
  Y. 40 mines on a 16 by 16 grid
  Z. 99 mines on a 30 by 16 grid
  Explain your reasoning.

Pupils should be taught to:

Use the probability scale; find and justify theoretical probabilities

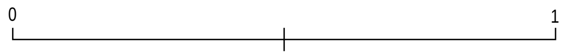
As outcomes, Year 7 pupils should, for example:

Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts.

Recognise that, for a finite number of possible outcomes, probability is a way of measuring the chance or likelihood of a particular outcome on a scale from 0 to 1, with the lowest probability at zero (impossible) and the highest probability at 1 (certain). For example:

- What fractions would you use to describe:
  a. the chance of picking a red card at random from a pack of 52 cards?
  b. the chance of picking a club card?

  Position the fractions on this probability scale.



Know that probability is related to proportion and can be represented as a fraction, decimal or percentage, e.g. discuss what is meant by a weather forecast of a 20% chance of rain.

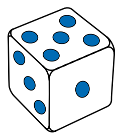
Know that if several equally likely outcomes are possible, the probability of a particular outcome chosen at random can be measured by:

$$\frac{\text{number of events favourable to the outcome}}{\text{total number of possible events}}$$

For example:

- The letters in the word **RABBIT** are placed in a tub, and a letter taken at random. What is the probability of taking out:
  a. a letter **T**? (*one in six, or* $\frac{1}{6}$)
  b. a letter **B**? ($\frac{2}{6}$ *or* $\frac{1}{3}$)

- The probability of rolling a 2 on a fair 1 to 6 dice is $\frac{1}{6}$, because 2 occurs once out of a total of 6 different possibilities.



  What is the probability of rolling:
  a. 5?
  b. an odd number?
  c. zero?
  d. a number greater than 2?
  e. a prime number?
  f. a number lying between 0 and 7?

  Mark these probabilities on a probability scale.

- A newsagent delivers these papers, one to each house.

Sun	250	*Times*	120
Mirror	300	*Mail*	100
Telegraph	200	*Express*	80

  What is the probability that a house picked at random has:
  a. the *Times*?
  b. the *Mail* or the *Express*?
  c. neither the *Sun* nor the *Mirror*?

Link to problems involving probability (pages 22–3).

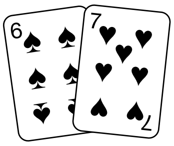
As outcomes, Year 8 pupils should, for example:

Know that if the probability of an event occurring is *p*, then the probability of it not occurring is 1 – *p*.

Use this to solve problems. For example:

- Consider a pack of 52 playing cards (no jokers). If the probability of drawing a club from a pack of cards is ¼, then the probability of drawing a card that is not a club is 1 – ¼, or ¾.

  Calculate the probability that a card chosen at random will be:
  a. a red card;
  b. a heart;
  c. not a picture;
  d. not an ace;
  e. either a club or a diamond;
  f. an even numbered red card.



- There are 25 cars parked in a garage. 12 are red, 7 blue, 3 white and the rest black. Calculate the probability that the next car to leave the garage will be:
  a. red;
  b. blue;
  c. neither red nor blue;
  d. black or white.

- A set of snooker balls consists of 15 red balls and one each of the following: yellow, green, brown, blue, pink, black and white. If one ball is picked at random, what is the probability of it being:
  a. red?
  b. not red?
  c. black?
  d. not black?
  e. black or white?

- Imrad threw a dart at a dartboard 60 times. Each time the dart hit the board. The maximum score for one dart is treble twenty. Imad scored treble twenty 12 times.

  Imrad is going to throw the dart once more. Estimate the probability that:
  a. he will score treble twenty;
  b. he will score less than 60.
  Give your reasons.

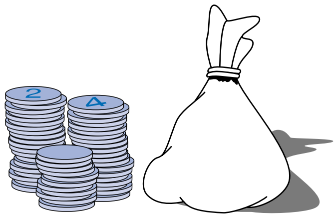
Link to problems involving probability (pages 22–3).

As outcomes, Year 9 pupils should, for example:

Know that the sum of probabilities of all mutually exclusive outcomes is 1.

Use this to solve problems. For example:

- A number of discs are placed in a bag.



  Most are marked with a number 1, 2, 3, 4 or 5. The rest are unmarked. The probabilities of drawing out a disc marked with a particular number are:

  p(1) = 0.15
  p(2) = 0.1
  p(3) = 0.05
  p(4) = 0.35
  p(5) = 0.2

  What is the probability of drawing a disc:
  a. marked 1, 2 or 3?
  b. not marked with a number?

- In an arcade game only one of four possible symbols can be seen in the final window. The probability of each occurring is:

Symbol	Probability
jackpot	1⁄16
moon	¼
star	?
lose	½

  a. What is the probability of getting a star?
  b. What event is most likely to happen?
  c. What is the probability of not getting the jackpot?
  d. After many games, the jackpot had appeared 5 times. How many games do you think had been played?

Link to problems involving probability (pages 22–3).

Pupils should be taught to:

Understand and use the probability scale; find and justify theoretical probabilities (continued)

As outcomes, Year 7 pupils should, for example:

Identify all the possible outcomes of a single event.

For example:

- What are the possible outcomes…

  a. when a fair coin is tossed?
  There are two outcomes: heads or tails.
  The probability of each is $\frac{1}{2}$.

  b. when a letter of the alphabet is chosen at random?
  There are two outcomes: a vowel or a consonant.
  The probability of a vowel is $\frac{5}{26}$.
  The probability of a consonant is $\frac{21}{26}$.

  c. when a letter from the word HIPPOPOTAMUS is picked at random?
  There are nine outcomes: H, I, P, O, T, A, M, U, S.
  The probability of H, I, T, A, M, U or S is $\frac{1}{12}$.
  The probability of O is $\frac{2}{12}$ or $\frac{1}{6}$.
  The probability of P is $\frac{3}{12}$ or $\frac{1}{4}$.

  d. when a number is chosen at random from the set of numbers 1 to 30?
  There are two outcomes:
  prime ($\frac{11}{30}$) or non-prime ($\frac{19}{30}$).
  or:
  There are two outcomes:
  odd ($\frac{15}{30}$ or $\frac{1}{2}$) or even ($\frac{15}{30}$ or $\frac{1}{2}$).
  or:
  There are three outcomes:
  a number from 1–10 ($\frac{10}{30}$ or $\frac{1}{3}$),
  a number from 11–20 ($\frac{10}{30}$ or $\frac{1}{3}$),
  a number from 21–30 ($\frac{10}{30}$ or $\frac{1}{3}$).
  and so on.

Link to problems involving probability (pages 22–3).

As outcomes, Year 8 pupils should, for example:

Find and record all possible outcomes for single events and two successive events in a systematic way, using diagrams and tables.

For example:

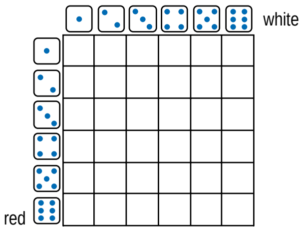
- A coin can land in two ways: head up (H) or tail up (T).

  Throw a coin twice. Record the four possible ways that the coin can land in two throws.

H	H
H	T
T	H
T	T

- What are the possible outcomes when:
  a. a mother gives birth to twins?
  b. a glazier puts red, green or blue glass in each of two windows?
  c. you can choose two pizza toppings from onion, mushroom and sweetcorn?

- One red and one white dice are numbered 1 to 6. Both dice are thrown and the scores added. Use a sample space to show all possible outcomes.



  Which score is the most likely? Why?

  Using the sample space, what is the probability of:
  a. getting the same number on both dice?
  b. the sum of the numbers being less than 4?
  c. the score on the red dice being double the score on the white dice?

- 200 raffle tickets are numbered from 1 to 200. They have all been sold. One ticket will be drawn at random to win first prize.
  a. Karen has number 125. What is the probability that she will win?
  b. Andrew buys tickets with numbers 81, 82, 83, 84. Sue buys tickets numbered 30, 60, 90, 120. Who has the better chance of winning? Why?
  c. Rob buys several tickets. He has a 5% chance of winning. How many tickets has he bought?
  d. Three people have each lost a ticket and do not play. What is the chance that nobody wins?

Link to problems involving probability (pages 22–3).

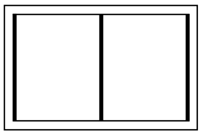
As outcomes, Year 9 pupils should, for example:

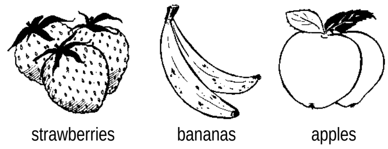
Identify all the mutually exclusive outcomes of an experiment.

For example:

- A fair coin and a fair dice are thrown. One possible outcome is (tail, 5). List all the other possible outcomes.

- A fruit machine has two 'windows'. In each window, one of three different fruits is equally likely to appear.



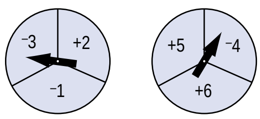


  List all the possible outcomes.

  What is the probability of getting:
  a. two identical fruits?
  b. at least one banana?
  c. no bananas?

- Two coins are thrown at the same time. There are four possible outcomes:
  HH HT TH TT
  How many possible outcomes are there if:
  a. three coins are used?
  b. four coins are used?
  c. five coins are used?

- The hands on these two spinners are spun at the same time.



  The two scores are added together. What is the probability that the total score is negative?

- A game involves rolling 6 dice. If you get 6 sixes you win a mountain bike. What is your chance of winning the bike?

Link to problems involving probability (pages 22–3).

Pupils should be taught to:

Collect and record experimental data, and estimate probabilities based on the data

As outcomes, Year 7 pupils should, for example:

Collect data from a simple experiment and record in a frequency table; estimate probabilities based on the data.

For example:

- Put four different coloured cubes in a bag. Shake it.
  Without looking, take a cube from the bag, but before you do so, guess its colour.
  If you are right, put a tick in the first column.
  If you are wrong, put a cross.
  Put the cube on the table.
  Carry on until you have taken out all four cubes.

  Repeat this experiment 10 times. Record your results.

Experiment number	Guesses			
	1st	2nd	3rd	4th
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

  What is the chance of being right on the 1st guess?
  On the 4th guess? Choose from: no chance, some chance, even chance, certain chance. Explain your choice.

- Use a bag containing an unknown mixture of identical, but differently coloured, counters.
  Draw one counter from the bag, note its colour, then replace it. Do this 10 times.
  Now estimate the probability of each colour.
  Check by emptying the bag.

- Make a dice (or spinner) from card in the shape of a regular solid or polygon. Weight it to make it biased, e.g. with Plasticine stuck to the inside or to the surface of the spinner.
  Throw the dice or spin the spinner 50 times.
  Estimate the probability of each score.
  Compare your estimated probabilities with what you would expect from a fair dice or spinner.

As outcomes, Year 8 pupils should, for example:

Estimate probabilities based on experimental data and use relative frequency as an estimate of probability. For example:

- Class 8C opened 20 small boxes of raisins.
  8 of the 20 boxes contained more than 28 raisins.
  What is the probability that an unopened box will contain fewer than 28 raisins?

- Throw two dice numbered 1 to 6 and sum the scores. Repeat 30 times and record each result on a frequency diagram.
  Compare results with another group.
  Are they different? Why?
  Predict what might happen if the experiment were repeated.
  Carry out the experiment again another 20 times and record the extra scores on the same diagram. What effect do the extra throws have on the results?
  Did the results match predictions? Why?

Understand that:
- If an experiment is repeated there may be, and usually will be, different outcomes.
- Increasing the number of times an experiment is repeated generally leads to better estimates of probability.

Solve problems such as:

- A girl collected the results of 50 European football matches:

home wins	35
away wins	5
draws	10

  Use these results to estimate the probability in future European matches of:
  a. a home win;
  b. an away win;
  c. a draw.

  The girl found the results of the next 50 matches.

home wins	37
away wins	4
draws	9

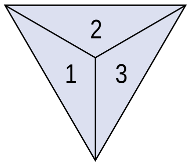
  Estimate, using all 100 results, the probability in future European matches of:
  a. a home win;
  b. an away win;
  c. a draw.
  Would these probabilities be more accurate than those based on the first 50 matches? Why?

Link to comparing experimental and theoretical probabilities (pages 284–5).

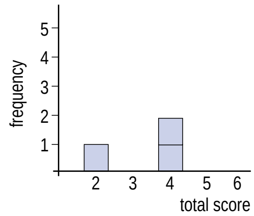
As outcomes, Year 9 pupils should, for example:

Estimate probabilities based on experimental data and use relative frequency as an estimate of probability. For example:

- Use an equilateral triangular spinner with three equal sections labelled 1, 2 and 3.
  Spin it twice. Add the two scores.
  Repeat this 40 times.



  As the experiment progresses, record results in a frequency diagram.

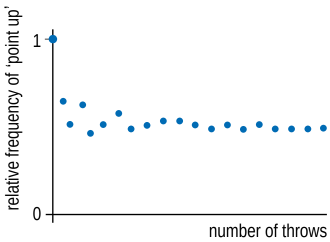


  Using the results:
  a. Which total is most likely?
  b. What is the estimated probability of a total of 5? How could you make a more accurate estimate?
  c. If the experiment were repeated 2000 times, how many times would you expect to get a total of 3?
  Justify your answers.

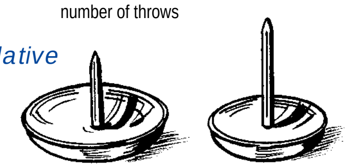
  What happens if the numbers on the spinner are changed, e.g. to 1, 2, 2 or 3, 2, 2 or 1, 1, 3 or 2, 2, 2?

Recognise that, with repeated trials, experimental probability tends to a limit. Relative frequency can give an estimate of probability, independent of the theoretical probability, and may be the only realistic way of estimating probability when events are not equally likely. For example:

- *Throw a drawing pin 10 times.*
  Record how many times it lands 'point up'.
  Estimate the probability of 'point up' from these 10 trials.
  Repeat the experiment another 10 times. Estimate the probability based on 20 trials.
  Repeat the procedure another 80 times, calculating and plotting the probabilities (relative frequencies) after every 10 throws.



Predict and sketch relative frequency diagrams for these pins.



Pupils should be taught to:

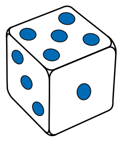
Compare experimental and theoretical probabilities

As outcomes, Year 7 pupils should, for example:

Compare experimental and theoretical probabilities in simple contexts.

For example:

- Use **computer software**, or a simple program using the RANDOM function, to simulate simple experiments, e.g. with dice or coins.



Compare these with the calculated results based on theoretical probabilities.
For example, simulate throwing a single dice a specified number of times. Observe that:
- if the experiment is repeated, the frequencies of different scores will vary;
- if the number of trials is large, then the frequencies settle down, each to approximately one sixth.

Answer questions such as:

- Jack, Lynn and Richard each rolled a fair dice 240 times. Here are the results they gave their teacher. One of them had recorded their results accurately but the other two had not.

Number	Jack	Lynn	Richard
1	42	40	26
2	37	40	33
3	48	40	27
4	36	40	96
5	36	40	34
6	41	40	24

Whose results were recorded accurately?
Explain your reasons.

As outcomes, Year 8 pupils should, for example:

Compare experimental and theoretical probabilities in different contexts.

For example:

- Use **computer software** to simulate throwing two dice, recording the total score and representing different outcomes on a frequency diagram. Observe:
  a. the pattern of results, relating these to thoretical frequencies (probabilities), based on an analyisis of combinations of scores;
  b. that increasing the number of trials produces a diagram which is closer to a triangular shape.

Answer questions such as:

- Michael said: 'I bet if I drop this piece of toast, it will land jam side down!'
  What is the theoretical probability of this outcome?
  Devise an experiment to test Michael's prediction (e.g. simulate the toast with a playing card – the jam is represented by the side with spots).
  Carry out the experiment.
  Compare the experimental and theoretical probabilities.

- Yasmin bought a combination padlock for her school locker.
  The code has four digits, each from 0 to 9.
  Yasmin forgot the last digit of the code.
  What is the theoretical probability that she will choose the correct digit first time?
  Design and carry out an experiment to estimate the experimental probability.
  Compare the outcome with the theoretical probability.

- The 49 balls in the National Lottery draw are numbered from 1 to 49.
  What is the probability of the first ball:
  a. being a multiple of 5?
  b. being odd?
  c. being a prime number?
  d. not containing the digit 1?
  Design an experiment to test these theoretical probabilities.
  You could refer to the **website** to find out historical data: www.lottery.co.uk

As outcomes, Year 9 pupils should, for example:

Compare experimental and theoretical probabilities in a range of contexts, appreciating the difference between mathematical explanation and experimental evidence. For example:

- Use **computer software** to simulate tossing a coin, plotting points to show the relative frequency of heads against the number of trials. Observe that:
  a. the pattern of results is erratic at first, but settles eventually to a value of approxiamtely ½;
  b. when the experiment is repeated, a similar pattern of results is observed.

- Explore games that may or may not be 'fair'.
  For example:

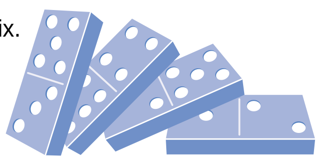
  Roll two dice 36 times. Add the two scores.
  If the outcome is EVEN, you WIN.
  If the outcome is ODD, you LOSE.

  Repeat the experiment, multiplying the two scores, rather than adding them.

  Does each game give the same probability of winning? Explain your reasons.
  Use a sample space diagram to justify results.

  What would you expect for results of EVEN, ODD and ZERO if you subtracted the two scores?

- Use a set of 28 dominoes, double blank to double six. Draw out one at a time from a bag. Record the total score and replace. Repeat many times.



  Construct a diagram to show the frequency of each score. Compare this with the diagram you would expect in theory.
  How and why is the distribution of the total of the numbers on a domino different from the totals of two dice?

- Throw two dice. Players chose one rule each and explain why they have chosen it. If the rule is satisfied the player gains a point. Predict then test the results after 20 throws.

Suggested rules for the two scores:

A	the difference is zero
B	the total is more than 8
C	the total is a factor of 12
D	the difference is 1
E	the total is prime
F	the total is a multiple of 3
G	the product is even
H	the two numbers share a factor

Think up and test your own rules.