Pupils should be taught to:

Generate and describe sequences

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
sequence, term, nth term, consecutive, rule, relationship, generate, predict, continue… increase, decrease… finite, infinite…

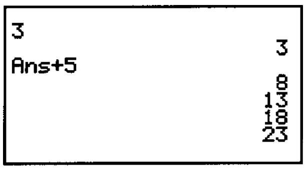
Know that:
- A number sequence is a set of numbers in a given order.
- Each number in a sequence is called a term.
- Terms next to each other are called consecutive and are often separated by commas, e.g. 6, 8, 10 and 12 are consecutive terms in a sequence of even numbers.

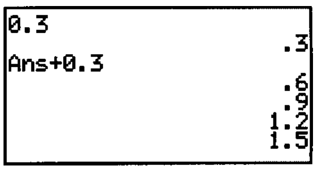
Know that a sequence can have a finite or an infinite number of terms, and give simple examples. For example:
- The sequence of counting numbers, 1, 2, 3, 4, 5, …, is infinite; dots indicate that counting continues indefinitely.
- The sequence of two-digit even numbers, 10, 12, 14, …, 98, is finite; dots indicate that the sequence continues in the same way until the final value 98 is reached.

Give simple examples of number sequences that:
- follow a simple rule that is easy to describe (e.g. odd numbers on house doors, square numbers);
- follow a more complex rule (e.g. time of sunset each day);
- follow an irregular pattern, affected by different factors (e.g. the maximum temperature each day);
- consist of a random set of numbers (e.g. numbers in the lottery draw).

Explore and predict terms in sequences generated by counting in regular steps, e.g. describe the sequences in words then use a simple **computer program** or **graphical calculator** to generate them and similar sequences.
Extend to decimals and negative numbers. For example:
- 8, 16, 24, 32, 40, …
- 5, 13, 21, 29, 37, …
- 89, 80, 71, 62, 53, …

- Start at 0 and count on in steps of 0.5. 0.5, 1, 1.5, 2, …
  Each term is a multiple of 0.5.
- Start at 41 and count back in steps of 5. 41, 36, 31, 26, …
  Each term is a multiple of 5 plus 1.





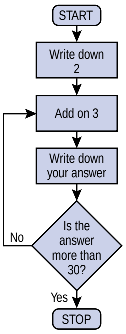
Begin to categorise familiar types of sequence. For example:
- Sequences can be ascending (the terms get bigger), or descending (the terms get smaller).
- Some sequences increase or decrease by equal steps.
- Some sequences increase or decrease by unequal steps.

See Y456 examples (pages 16–17).

As outcomes, Year 8 pupils should, for example:

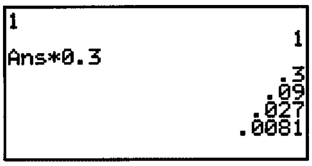
Use vocabulary from previous year and extend to:
difference pattern, general term, T(n)...
flow chart... linear sequence, arithmetic sequence...

Generate sequences from **flow charts**. For example:



Continue familiar sequences.
For example:
- Square numbers 1, 4, 9, 16, …
- Powers of 10 10, 100, 1000, 10 000, …
- Powers of 2 64, 32, 16, …

Generate sequences by multiplying or dividing by a constant factor. For example:
- 1, $^{-}2$, 4, $^{-}8$, 16, $^{-}32$, …
- 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, …
- 1, 0.5, 0.25, 0.125, …



Generate sequences by counting forwards or backwards in increasing or decreasing steps.
For example:
- Start at 30 and count forwards by 1, 2, 3, … to give 31, 33, 36, …
- Start at 1 and count forwards by 2, 3, 4, … to give the triangular numbers 1, 3, 6, 10, …

Know that, unless a rule is specified, sequences may not continue in the most obvious way and 'spotting a pattern' may lead to incorrect conclusions.
For example:
- Explain why 1, 2, 4, … may continue 1, 2, 4, 8, 16, … or 1, 2, 4, 7, 11, … or in other ways.
- Explain why 1, 2, 3, … may continue 1, 2, 3, 4, 5, … or 1, 2, 3, 5, 8, … or in other ways.

Begin to appreciate that:
- Seeing a pattern in results enables predictions to be made, but any prediction must always be checked for correctness.
- A satisfactory conclusion is reached only when a general explanation of either a term-to-term or a position-to-term rule is given and the pattern can be justified.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
quadratic sequence...
first difference, second difference...

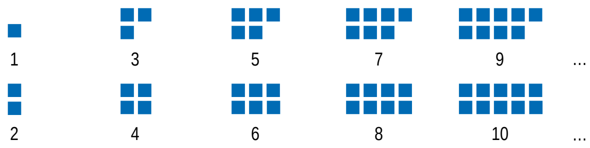
Pupils should be taught to:

Generate and describe sequences (continued)

As outcomes, Year 7 pupils should, for example:

Generate and describe simple integer sequences and relate them to geometrical patterns. For example:

- Odd and even numbers

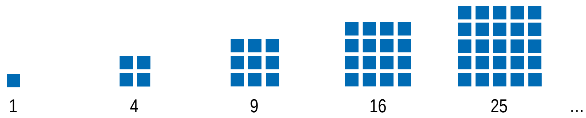


Know that:
Odd numbers are so called because, when arranged in pairs, there is always an odd one left unmatched.
Even numbers are so called because they can be arranged in matched pairs.

- Multiples of 3

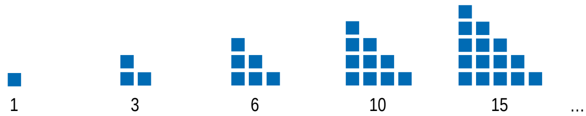
Know that multiples of 3 can be arranged in rectangular patterns of width 3.

- Square numbers



Know that square numbers make square patterns of dots and that 1 is counted as the first square number.

- Triangular numbers



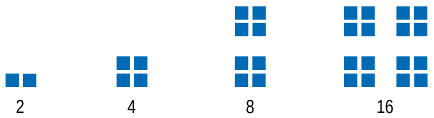
Know that triangular numbers can be arranged in triangular patterns of dots and that 1 is counted as the first triangular number.

Link to work on integers, powers and roots (pages 48–59).

As outcomes, Year 8 pupils should, for example:

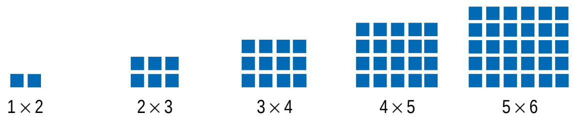
Generate and describe integer sequences, relating them to geometrical patterns. For example:

- Powers of 2



Visualise powers of 2 as:
a row of 2 dots,
a square of 4 dots,
a column of two 4-dot squares,
a square of four 4-dot squares…

- Growing rectangles



Classify familiar sequences according to whether they are ascending or descending and whether they do so by equal or unequal steps.

Know that an arithmetic sequence is generated by starting with a number a, then adding a constant number d to the previous term. For example:

- If $a = 2$ and $d = 3$, the sequence is
  2, 5, 8, 11, 14, …
- If $a = 4$ and $d = {}^{-}2$, the sequence is
  4, 2, 0, $^{-}2$, $^{-}4$, …

Link to work on integers, powers and roots (pages 48–59).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT

As outcomes, Year 7 pupils should, for example:

Generate terms of a sequence given a rule for finding each term from the previous term, on paper and using **ICT**.
For example:

- Generate the first few terms of these sequences and then describe them in words:

<u>1st term</u>	<u>Term-to-term rule</u>
10	add 3
	Each term is 3 more than the one before.
100	subtract 5
	Each term is 5 less than the one before.
2	double
	Each term is double the one before.
5	multiply by 10
	Each term is 10 times the one before.

- Here is a rule for a sequence: *To find the next term add 3.*
  There are many sequences with this rule.
  Is it possible to find one for which:
  a. all the numbers are multiples of 3?
  b. all the numbers are odd?
  c. all the numbers are multiples of 9?
  d. none of the numbers is a whole number?
  Explore with a **graphical calculator**.

Generate a sequence given a rule for finding each term from its position in the sequence. For example:

- The nth term of a sequence is $n + 3$. Write the first five terms.
- The nth term of a sequence is:
  a. $n + 7$ b. $105 – 5n$ c. $2n – 0.5$ d. $4n$
  Write the first five terms of each sequence.
  Describe each sequence in words (e.g. odd numbers), or using a rule for generating successive terms (e.g. first term is 1, rule is ‘add 2’).

Recognise that sequences of multiples can be generated in two ways:

- They can be generated by using a term-to-term rule of repeated addition of the number, e.g. for multiples of 6:
  6, 6 + 6, 6 + 6 + 6, 6 + 6 + 6 + 6, …
  1st term = 6 Term-to-term rule ‘add 6’
  The difference between consecutive terms is always 6.

	A	B	C	D	E	F
1	Position	1	2	3	4	5
2	Term	6	=B2+6	=C2+6	=D2+6	=E2+6

- Each multiple can be calculated from its position, using a position-to-term rule:

1st term	2nd term	3rd term	…	nth term
1×6	2×6	3×6	…	$n \times 6$ (or $6n$)

	A	B	C	D	E	F
1	Position	1	2	3	4	5
2	Term	=B1*6	=C1*6	=D1*6	=E1*6	=F1*6

The nth term can be found more quickly using a position-to-term rule, particularly for terms a long way into the sequence, such as the 100th term.

As outcomes, Year 8 pupils should, for example:

Generate terms of a sequence given a rule for finding each term from the previous term, on paper and using **ICT**. For example:

- Generate and describe in words these sequences.

<u>1st term(s)</u>	<u>Term-to-term rule</u>
8	subtract 4
1	add consecutive odd numbers, starting with 3
28	halve
1 000 000	divide by 10
1, 2, …	add the two previous terms

- Here is a rule to find the next term of a sequence:
  Add ☐.
  Choose a first term for the sequence and a number to go in the box in such a way to make all the terms of the sequence:
  a. even;
  b. odd;
  c. multiples of 3;
  d. all numbers ending in the same digit.
  Explore with a **graphical calculator**.

Generate a sequence given a rule for finding each term from its position in the sequence, referring to terms as T(1) = first term, T(2) = second term, …, T(n) = nth term. For example:

- The nth term of a sequence is $2n$, i.e. $T(n) = 2n$. Write the first five terms.

- Write the first five terms of a sequence whose nth term or T(n) is:
  a. $5n + 4$ c. $99 - 9n$ e. $3n - 0.1$
  b. $105 - 5n$ d. $n + ½$ f. $n \times 0.1$

Use a **spreadsheet** or **graphical calculator** to find particular terms such as:
- the 24th multiple of 13
  in the sequence 13, 26, 39, …
- the 100th multiple of 27
  in the sequence 27, 54, 71, …

- Write the nth multiple of 18
  in the sequence 18, 36, 54, …

As outcomes, Year 9 pupils should, for example:

Generate terms of a sequence given a rule for finding each term from the previous term, on paper and using **ICT**. For example:

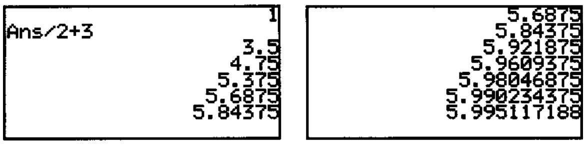
Review general properties of linear sequences of the form $an + b$, where a and b take particular values, e.g. $2n + 5$, $3n - 7$, $10 - 4n$.

- The sequence can also be defined by
  1st term, $T(1) = a + b$
  term-to-term rule, 'add a to the previous term'
  so $T(2) = a + T(1)$, etc.
- If the constant difference a between successive terms is positive, the sequence is ascending;
  if a is negative, the sequence is descending.

- Here is a rule for a sequence:
  To find the next term of the sequence add ☐.
  Choose a first term for the sequence and a number to go in the box in such a way that:
  a. every other number is an integer;
  b. every fourth number is an integer;
  c. there are exactly ten two-digit numbers in the sequence;
  d. every fourth number is a multiple of 5.
  Explore with a **graphical calculator**.

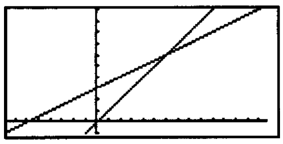
- *The nth term of a sequence is given by*
  $T(n) = 2n + (n-1)(n-2)(n-3)(n-4)$
  Write down the first five terms of the sequence.

- *The sequence 'divide by 2, add 3', starting with 1, can be represented as* $x \rightarrow \frac{x}{2} + 3$.



Link the apparent limiting value of 6 to the solution of the equation $x = \frac{x}{2} + 3$

and to the intersection of the graphs of $y = x$ *and* $y = \frac{x}{2} + 3$



Explore the effect of varying the numbers 2 and 3 on the limiting value.

Pupils should be taught to:

Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence (continued)

As outcomes, Year 7 pupils should, for example:

Use a **spreadsheet** to generate tables of values and explore **term-to-term** and **position-to-term** relationships.
For example:

- The 1st term is 100, and the rule is 'subtract 5'.

	A	B	C	D	E	F	G	H
1	Position	1	=B1+1	=C1+1	=D1+1	=E1+1	=F1+1	=G1+1
2	Term	100	=B2–5	=C2–5	=D2–5	=E2–5	=F2–5	=G2–5

	A	B	C	D	E	F	G	H
1	Position	1	2	3	4	5	6	7
2	Term	100	95	90	85	80	75	70

Arrange a sequence in a table. For example:

- Multiples of 3:

Position	1	2	3	4	5	…	n
Term	3	6	9	12	15	…	$3 \times n$

As outcomes, Year 8 pupils should, for example:

Use a **spreadsheet** to generate tables of values and explore **term-to-term** and **position-to-term** linear relationships. For example:

- The *n*th term is $3n + 7$.

	A	B
1	Position	Term
2	1	=A2*3+7
3	=A2+1	=A3*3+7
4	=A3+1	=A4*3+7
5	=A4+1	=A5*3+7
6	=A5+1	=A6*3+7
7	=A6+1	=A7*3+7
8	=A7+1	=A8*3+7
9	=A8+1	=A9*3+7
10	=A9+1	=A10*3+7

	A	B
1	Position	Term
2	1	10
3	2	13
4	3	16
5	4	19
6	5	22
7	6	25
8	7	28
9	8	31
10	9	34

Arrange a sequence in a table, referring to terms as T(1) = first term, T(2) = second term, …, T(*n*) = *n*th term. For example, for multiples of 3:

Position *n*	1	2	3	4	5	…	*n*
T(*n*)	3	6	9	12	15	…	T(*n*)
Difference	3	3	3	3		…	

Explain the effect on a sequence of multiples if a constant number is added to or subtracted from each term. For example:

- $T(n) = 2n + b$:
  $2n - 1$ generates the odd numbers, starting at 1, because each is one less than an even number.
  $2n + 1$ generates odd numbers, starting at 3.
  $2n + 2$ generates even numbers, starting at 4.
  $2n - 6$ generates even numbers, starting at $^{-}4$.

- $T(n) = 3n + b$:
  If *b* is a multiple of 3, this generates multiples of 3, starting at different numbers.
  Otherwise, it generates a sequence with a difference of 3 between consecutive terms.

- $T(n) = 10n + b$:
  If *b* is between 0 and 9, this generates numbers whose units digit is *b*.

Explain how descending sequences can be generated by subtracting multiples from a constant number: $T(n) = b - an$. For example:

- $T(n) = 6 - n$ generates descending integers:
  5, 4, 3, 2, 1, 0, $^{-}1$, $^{-}2$, $^{-}3$, $^{-}4$, $^{-}5$, …
- $T(n) = 110 - 10n$ generates the 10 times table backwards.

Explore difference patterns between terms of a linear sequence of the form $an + b$. Observe that the differences are constant and equal to *a*. Use this result when searching for a rule or for the *n*th term.

As outcomes, Year 9 pupils should, for example:

Begin to generate a quadratic sequence given a rule for finding each term from its position. For example:

- Generate the first ten terms of these sequences:
  $T(n) = n^2$ $T(n) = 2n^2 + 1$ $T(n) = n^2 - 2$

- The *n*th term of a sequence is $n/(n^2 + 1)$.
  The first term of the sequence is ½.
  Write the next three terms.
  Use a **spreadsheet** to generate terms beyond the third term.

*Use **ICT** to generate non-linear sequences. For example:*

- *Use a **computer program** to generate sequences where the second row of differences (i.e. the increment in the step) is constant, as in:*

	1st term	Initial step (1st difference)	Step increment (2nd difference)
a.	1	1	1
b.	58	$^{-}1$	$^{-}3$
c.	7	18	$^{-}2$
d.	100	$^{-}17$	5

- *Generate a familiar sequence, e.g. square or triangular numbers.*

- *Generate the first few terms of the sequence of square numbers, $T(n) = n^2$, and examine term-to-term difference patterns:*

  1 4 9 16 25 36 …
  3 5 7 9 11 …
  2 2 2 2 …

  Observe that the terms increase by successive odd numbers, and that the second row of differences in always 2.

- *Explore the sequences generated by these formulae for T(n), by finding some terms and examining difference patterns:*
  $3n^2 + 2$ $4 - 3n^2$ $n^2 + n$
  $n(n - 1)/2$ $4n - 6$ $n^2 + 4n - 6$

Know that rules of the form $T(n) = an^2 + bn + c$ generate quadratic sequences. Know that the second row of differences is constant and equal to 2a. Use this result when searching for a quadratic rule.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Find the next term and the nth term of a sequence where the rule is quadratic or closely related to $T(n) = n^2$.

Prove that, for a sequence of the form
$T(1) = a + b + c$, $T(n) = an^2 + bn + c$
the second differences between successive terms are constant and equal to 2a.

- *Find the nth term of the sequence*
  6, 15, 28, 45, 66, …
  ($T(1) = 6$, $T(n) = T(n-1) + 9 + 4(n-2)$)

  Pattern of differences:

6	15	28	45	66	91 …
9	13	17	21	25 …	
4	4	4	4		

  The second differences are 4, so the sequence is of the form $T(n) = an^2 + bn + c$, and $a = 2$.
  $T(1) = 6$, so $2 + b + c = 6$
  $T(2) = 15$, so $8 + 2b + c = 15$
  Subtracting T(1) from T(2) gives $6 + b = 9$, so $b = 3$ and $c = 1$.
  So $T(n) = 2n^2 + 3n + 1$.

- *Find the nth term of the sequence*
  2, 5, 10, 17, 26, …
  ($T(1) = 2$, $T(n) = T(n-1) + 3 + 2(n-2)$)

  What is the nth term if:
  a. $T(1) = 0$?   b. $T(1) = 9$?

Explore how some fraction sequences continue.
For example:

- *$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, …, 2^{-n}, …*
- *$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, …, $\frac{n}{(n+1)}$, …*
- *$\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{8}{5}$, …*
  (from the Fibonacci sequence)
- *A flea sits in middle of circular table of radius 1 m. It takes a series of jumps towards the edge of the table. On the first jump it jumps half way to the edge of the table. On each succeeding jump it jumps half the remaining distance.*
  Investigate the total distance travelled after 1, 2, 3, … jumps.
  How long will it take the flea to reach the edge of the table?

Pupils should be taught to:

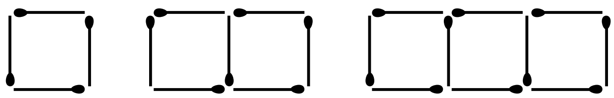
Find the *n*th term, justifying its form by referring to the context in which it was generated

As outcomes, Year 7 pupils should, for example:

Generate sequences from simple practical contexts.
- Find the first few terms of the sequence.
- Describe how it continues by reference to the context.
- Begin to describe the general term, first using words, then symbols; justify the generalisation by referring to the context.

For example:

- Growing matchstick squares

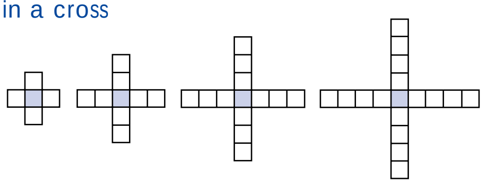


Number of squares	1	2	3	4	…
Number of matchsticks	4	7	10	13	…

Justify the pattern by explaining that the first square needs 4 matches, then 3 matches for each additional square, or you need 3 matches for every square plus an extra one for the first square.

In the *n*th arrangement there are $3n + 1$ matches.
Possible justification:
Every square needs three matches, plus one more for the first square, which needs four. For n squares, the number of matches needed is 3n, plus one for the first square.

- Squares in a cross



Size of cross	1	2	3	4	…
Number of squares	5	9	13	17	…

Justify the pattern by explaining that the first cross needs 5 squares, then 4 extra for the next cross, one on each 'arm', or start with the middle square, add 4 squares to make the first pattern, 4 more to make the second, and so on.

In the *n*th cross there are $4n + 1$ squares.

Possible justification:
The crosses have four 'arms', each increasing by one square in the next arrangement. So the nth cross has 4n squares in the arms, plus one in the centre.

- Making different amounts using 1p, 2p and 3p stamps

Amount	1p	2p	3p	4p	5p	6p	…
No. of ways	1	2	3	4	5	7	…

From experience of practical examples, begin to appreciate that some sequences do not continue in an 'obvious' way and simply 'spotting a pattern' may lead to incorrect results.

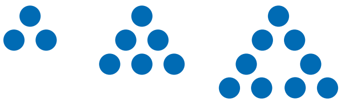
As outcomes, Year 8 pupils should, for example:

Generate sequences from practical contexts.

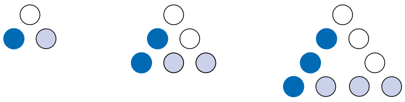
- Find the first few terms of the sequence; describe how it continues using a term-to-term rule.
- Describe the general (*n*th) term, and justify the generalisation by referring to the context.
- When appropriate, compare different ways of arriving at the generalisation.

For example:

- Growing triangles



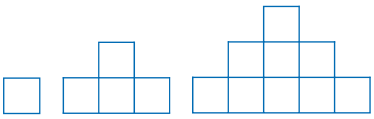
This generates the sequence: 3, 6, 9...
Possible explanations:
We add three each time because we add one more dot to each side of the triangle to make the next triangle.
It's the 3 times table because we get...



3 lots of 1   *3 lots of 2*   *3 lots of 3*   *etc.*

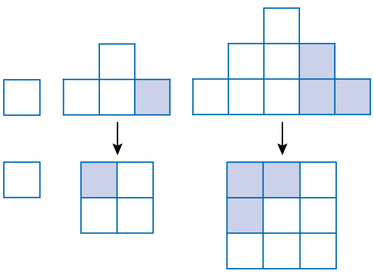
The general (*n*th) term is $3 \times n$ or $3n$.
Possible justification:
This follows because the 10th term would be '3 lots of 10'.

- 'Pyramid' of squares



This generates the sequence: 1, 4, 9, ...
Possible explanation:
The next 'pyramid' has another layer, each time increasing by the next odd number 3, 5, 7, ...

The general (*n*th) term is $n \times n$ or n^2.
Possible justification:
The pattern gives square numbers. Each 'pyramid' can be rearranged into a square pattern, as here:



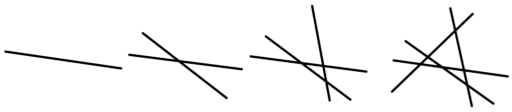
As outcomes, Year 9 pupils should, for example:

Generate sequences from practical contexts.

- Find the first few terms of the sequence; describe how it continues using a term-to-term rule.
- Use algebraic expressions to describe the *n*th term, justifying them by referring to the context.
- When appropriate, compare different ways of arriving at the generalisation.

For example:

- Maximum crossings for a given number of lines

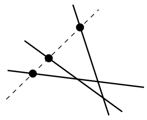


Number of lines	1	2	3	4	...
Maximum crossings	0	1	3	6	...

Predict how the sequence might continue and test for several more terms.

Discuss and follow an explanation, such as:

A new line must cross all existing lines.

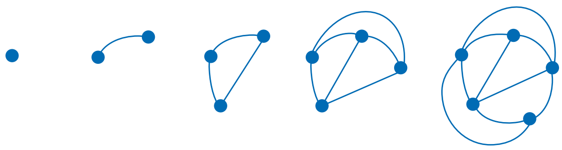


So when a new line is added, the number of extra crossings will equal the existing number of lines, e.g. when there is one line, an extra line adds one crossing, when there are two lines, an extra line adds two crossings, and so on.

No. of lines	1		2		3		4		5	...
Max. crossings	0		1		3		6		10	...
Increase		1		2		3		4		...

- Joining points to every other point

Joins may be curved or straight. Keeping to the rule that lines are not allowed to cross, what is the maximum number of joins that can be made?



No. of points	1	2	3	4	5	...
Maximum joins	0	1	3	6	9	...

Predict how the sequence might continue, try to draw it, discuss and provide an explanation.

Pupils should be taught to:

Find the *n*th term, justifying its form by referring to the context in which it was generated (continued)

As outcomes, Year 7 pupils should, for example:

Begin to find a simple rule for the *n*th term of some simple sequences.

For example, express in words the *n*th term of counting sequences such as:

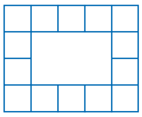
- 6, 12, 18, 24, 30, … *n*th term is six times *n*
- 6, 11, 16, 21, 26, … *n*th term is five times *n* plus one
- 9, 19, 29, 39, 49, … *n*th term is ten times *n* minus one
- 40, 30, 20, 10, 0, … *n*th term is fifty minus ten times *n*

Link to generating sequences using term-to-term and position-to-term definitions (pages 148–51).

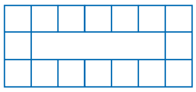
As outcomes, Year 8 pupils should, for example:

- Paving stones

  Investigate the number of square concrete slabs that will surround rectangular ponds of different sizes. Try some examples:



2 by 3 pond
needs 14 slabs



1 by 5 pond
needs 16 slabs

  Collect a range of different sizes and count the slabs. Deduce that, for an l by w rectangle, you need $2l + 2w + 4$ slabs.
  Possible justification:
  You always need one in each corner, so it's twice the length plus twice the width, plus the four in the corners.

  Other ways of counting could lead to equivalent forms, such as $2(l + 2) + 2w$ or $2(l + 1) + 2(w + 1)$.

  Confirm that these formulae give consistent values for the first few terms.

Check predicted terms for correctness.

Link to simplifying and transforming algebraic expressions (pages 116–19).

Begin to use linear expressions to describe the *n*th term of an arithmetic sequence, justifying its form by referring to the activity from which it was generated.

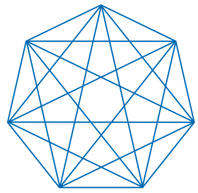
Develop an expression for the nth term of sequences such as:

- 7, 12, 17, 22, 27, … $\quad 2 + 5n$
- 100, 115, 130, 145, 160, … $\quad 15n + 85$
- 2.5, 4.5, 6.5, 8.5, 10.5, … $\quad (4n + 1)/2$
- $^-12$, $^-7$, $^-2$, 3, 8, … $\quad 5n – 17$
- 4, $^-2$, $^-8$, $^-14$, $^-20$, … $\quad 10 – 6n$

As outcomes, Year 9 pupils should, for example:

- Diagonals of polygons

  How many diagonals does a polygon have altogether (crossings allowed)?



No. of sides	1	2	3	4	5	6	7	…
No. of diagonals	–	–	0	2	5	9	14	…

  Explain that, starting with 3 sides, the terms increase by 2, 3, 4, 5, …

  Follow an explanation to derive a formula for the nth term, such as:
  Any vertex can be joined to all the other vertices, except the two adjacent ones.
  In an n-sided polygon this gives n – 3 diagonals, and for n vertices a total of n(n – 3) diagonals. But each diagonal can be drawn from either end, so this formula counts each one twice.
  So the number of diagonals in an n-sided polygon is ½n(n – 3).

  Confirm that this formula gives consistent results for the first few terms.

Know that for sequences generated from an activity or context:
- Predicted values of terms need to be checked for correctness.
- A term-to-term or position-to-term rule needs to be justified by a mathematical explanation derived from consideration of the context.

Use linear expressions to describe the *n*th term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated.

Find the nth term of any linear (arithmetic) sequence. For example:

- Find the nth term of 21, 27, 33, 39, 45, …

  The difference between successive terms is 6, so the nth term is of the form $T(n) = 6n + b$.
  $T(1) = 21$, so $6 + b = 21$, leading to $b = 15$.
  $T(n) = 6n + 15$
  Check by testing a few terms.

- Find the nth term of these sequences:
  54, 62, 70, 78, 86, …
  68, 61, 54, 47, 40, …
  2.3, 2.5, 2.7, 2.9, 3.1, …
  $^-5$, $^-14$, $^-23$, $^-32$, $^-41$, …

Pupils should be taught to:

As outcomes, Year 7 pupils should, for example:

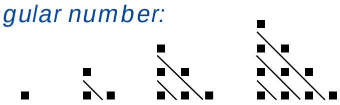
Find the *n*th term, justifying its form by referring to the context in which it was generated (continued)

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

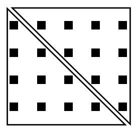
Explore spatial patterns for triangular and square numbers. *For example:*

- *Generate a pattern for specific cases of T(n), the nth triangular number:*

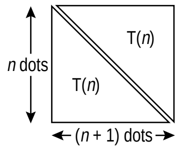


By considering the arrangement of dots, express T(n) as the sum of a series:
$T(n) = 1 + 2 + 3 + \ldots + n$

By repeating the triangular pattern to form a rectangle, deduce a formula for T(n):

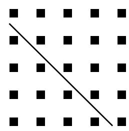


$T(4) + T(4)$
$= 4 \times 5$



$T(n) + T(n) = n(n + 1)$ or $T(n) = ½n(n + 1)$
Use this result to find the sum of the first 100 whole numbers: 1 + 2 + 3 + … + 100.

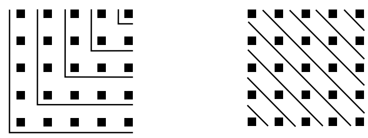
- *Split a square array of dots into two triangles:*



$T(4) + T(5) = 5^2$

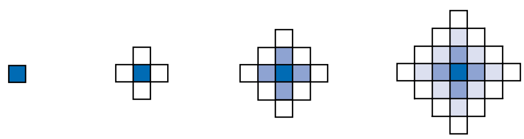
Deduce the result $T(n – 1) + T(n) = n^2$.
Test it for particular cases.

Consider other ways of dividing a square pattern of dots. For example:



Deduce results such as $1 + 3 + 5 + 7 + 9 = 5^2$.
Generalise to a formula for the sum of the first n odd numbers: $1 + 3 + 5 + \ldots + (2n – 1) = n^2$.
Say what can be deduced from the other illustration of dividing the square.

- *Certain 2-D 'creatures' start as a single square cell and grow according to a specified rule. Investigate the growth of a creature which follows the rule 'grow on sides':*



Stage 1	*Stage 2*	*Stage 3*	*Stage 4*	…
1 cell	*5 cells*	*13 cells*	*25 cells*	…

Investigate other rules for the growth of creatures.

Pupils should be taught to:

Express functions and represent mappings

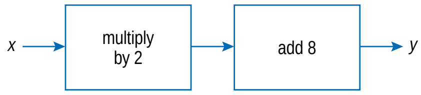
As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
input, output, rule, function, function machine, mapping…

Express simple functions at first in words then using symbols.
For example:

Explore simple function machines by:
- finding outputs (y) for different inputs (x);
- finding inputs for different outputs.

For example:



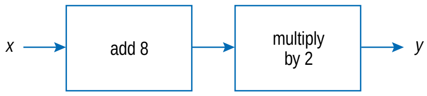
- What happens when the input is 5?
- What input gives an output of 40?

Descibe the effect of this function machine as $x \times 2 + 8 = y$.

Produce a table of inputs and outputs, such as:

x	1	2	3	4	5	6	7
y	10	12	14				

- What happens if the order of the machines is changed?



Descibe the effect of this function machine as $(x + 8) \times 2 = y$.

Participate in a function guessing game. For example:
- Find the secret rule that connects the blue number on the left to the black number on the right, e.g.
  (blue number – 1) × 2 = black number.

3 → **4**
7 → **12**
5 → **8**
9 → **16**

Participants in the game remain silent, but can:
A. offer a blue number, **or**
B. offer a black number for a given blue number, **or**
C. write in words what they think the rule is.

Know that the function which generates the output for a given input, can be expressed either by using a mapping arrow (→) or by writing an equation. For example:

blue number → (blue number – 1) × 2
or (blue number – 1) × 2 = black number
or black number = (blue number – 1) × 2

Draw simple mapping diagrams, e.g. for $x \to x + 2$.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
linear function…

Express simple functions in symbols.
For example:

Generate sets of values for simple functions using a function machine or a spreadsheet. For example:

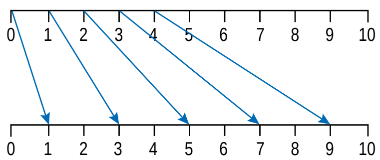
- Use a **spreadsheet** to produce a table of inputs and outputs, e.g. $x \rightarrow 2x + 8$ or $y = 2x + 8$.

	A	B
1	x	y
2	1	=A2*2+8
3	=A2+1	=A3*2+8
4	=A3+1	=A4*2+8
5	=A4+1	=A5*2+8
6	=A5+1	=A6*2+8
7	=A6+1	=A7*2+8
8	=A7+1	=A8*2+8

	A	B
1	x	y
2	1	10
3	2	12
4	3	14
5	4	16
6	5	18
7	6	20
8	7	22

Extend to negative and non-integral values.

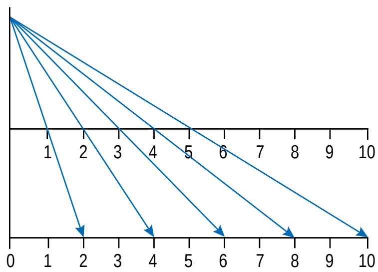
Draw mapping diagrams for simple functions.
For example, $x \rightarrow 2x + 1$:



Extend the mapping to include:
- negative integers down to $^{-}10$;
- fractional values.

Know some properties of mapping diagrams.
For example:

- Functions of the form $x \rightarrow x + c$ produce sets of parallel lines.
- Mapping arrows for multiples, if projected backwards, meet at a point on the zero line, e.g. $x \rightarrow 2x$:

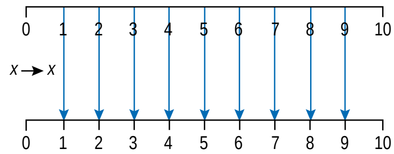


Link to enlargement by a whole-number scale factor (pages 212–13).

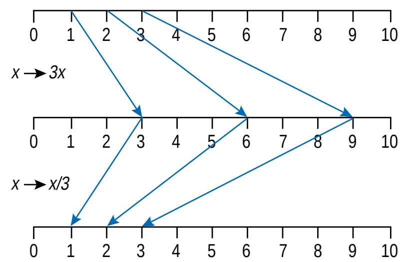
As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
identity function, inverse function, quadratic function… inverse mapping… self-inverse…

Know that $x \rightarrow x$ is called the identity function, because it maps any number on to itself, i.e. leaves the number unchanged.



Know that every linear function has an inverse function which reverses the direction of the mapping. For example, the inverse of multiplying by 3 is dividing by 3, and this can be expressed in symbols: the inverse of $x \rightarrow 3x$ is $x \rightarrow x/3$.

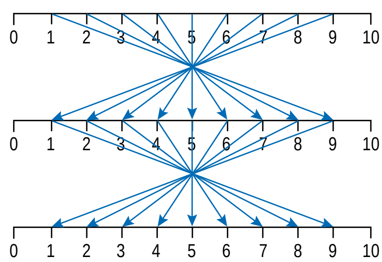


Find the inverse of a linear function such as:

- $x \rightarrow 3x + 1$
- $x \rightarrow 5x - 4$
- $x \rightarrow 2(x - 7)$
- $x \rightarrow \dfrac{x + 8}{10}$
- $x \rightarrow \frac{1}{4}x - 5$
- $x \rightarrow \frac{1}{2}x + 20$

Know that functions of the form $x \rightarrow c - x$ are self-inverse. For example:

- The inverse of $x \rightarrow 10 - x$ is $x \rightarrow 10 - x$.



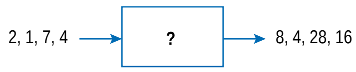
Pupils should be taught to:

Express functions and represent mappings (continued)

As outcomes, Year 7 pupils should, for example:

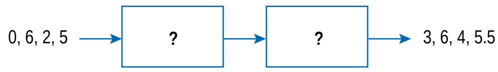
Given inputs and outputs, find the function.
For example:

- Find the rule (single machine):

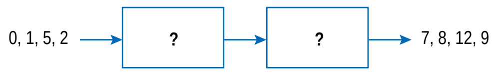


Multiply the input by 4 or, in symbols, $x \rightarrow 4x$.

- Find the rule (double machine):



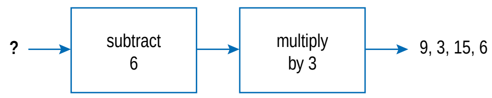
Divide by 2 and add 3, or $x \rightarrow x/2 + 3$.



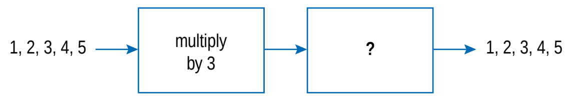
Different solutions are possible – and the two functions can be replaced by a single function.

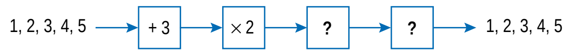
Explore inverse operations to find the input given the output.

- Given the output, find the input for a particular machine:



- Define the other machine(s):





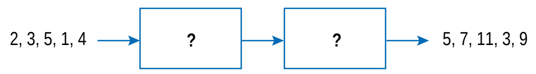
Begin to recognise some properties of simple functions.

- A function can sometimes be expressed in more than one way,
  e.g. red number → (red number – 1) × 2
  or red number → red number × 2 – 2
- A function can sometimes be expressed more simply, e.g.
  red number → red number × 3 × 5
  can be simplified to
  red number → red number × 15
- A function can often be inverted, e.g.
  if (red number – 1) × 2 = green number
  then green number ÷ 2 + 1 = red number

Link to inverse operations, equations and formulae (pages 114–15).

As outcomes, Year 8 pupils should, for example:

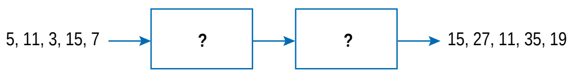
Given inputs and outputs, find the function. Given a linear function, put random data in order and use difference patterns to help find the function.
For example, find the rule:

- 

  Reorganise the data:

Input (x)	1		2		3		4		5
Output (y)	3		5		7		9		11
Difference		2		2		2		2	

  Recognise differences of 2. Try $x \to 2x + c$.
  From the first entry, find that $c = 1$.
  Check other values.

- 

  Reorganise the data:

Input (x)	3		5		7		11		15
Output (y)	11		15		19		27		35
Difference		4		4		8		8	

  Recognise that the first two differences are 4, where x is increasing by 2 each time.
  Try $x \to 2x + c$. From the first entry, find that $c = 5$.
  Check other values.

Link linear functions to linear sequences, particularly difference patterns (pages 148–51).

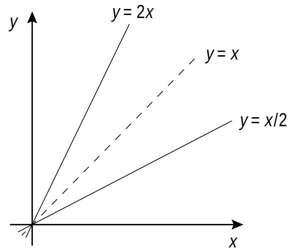
Know some properties of functions produced by combining number operations. For example:

- Two additions, two subtractions, or an addition with a subtraction, will simplify to a single addition or subtraction.
- Two multiplications, two divisions, or a multiplication with a division, will simplify to a single multiplication or division.
- A function may often be expressed in more than one way, e.g.
  $x \to 2x - 2$ is equivalent to $x \to 2(x - 1)$.
- Changing the order of two operations will often change the function, e.g.
  $x \to 3x - 4$ is different from $x \to 3(x - 4)$.
- The inverse of two combined operations is found by inverting the operations and reversing the order, e.g.
  the inverse of $x \to 2(x - 1)$ is $x \to x/2 + 1$.

Link to inverse operations, equations and formulae (pages 114–15).

As outcomes, Year 9 pupils should, for example:

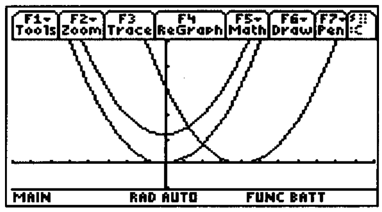
Plot the graph of a linear function, together with its inverse, *on paper or using* ***ICT****. For example:*



Observe the relationship between the two graphs: each is the reflection of the other in the line y = x.

Know some properties of quadratic functions *and features of their graphs. For example:*

- *The graph is a curve, symmetrical about the vertical line through its turning point.*
- *The value of the y-coordinate at the turning point is either a maximum or a minimum value of the function.*



Link to properties of quadratic sequences (pages 152–3), and plotting graphs of simple quadratic and cubic functions (pages 170–1).