Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Understand and use the language and notation associated with reflections, translations and rotations

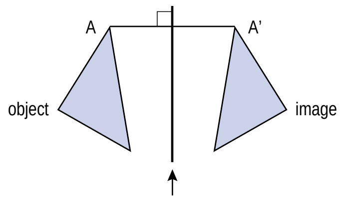
Use, read and write, spelling correctly:
transformation… image, object, congruent…
reflection, mirror line, line of symmetry, line symmetry, reflection symmetry, symmetrical…
translation… rotate, rotation, rotation symmetry, order of rotation symmetry, centre of rotation…

Recognise and visualise transformations and symmetries of shapes

Reflection

Understand reflection in two dimensions as a transformation of a plane in which points are mapped to images in a mirror line or axis of reflection, such that:

- the mirror line is the perpendicular bisector of the line joining point A to image A';
- the image is the same distance behind the mirror as the original is in front of it.



Know that a reflection has these properties:

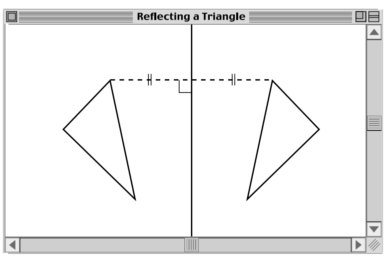
- Points on the mirror line do not change their position after the reflection, i.e. they map to themselves.
- A reflection which maps A to A' also maps A' to A, i.e. reflection is a self-inverse transformation.

Relate reflection to the operation of folding. For example:

- Draw a mirror line on a piece of paper. Mark point P on one side of the line. Fold the paper along the line and prick through the paper at point P. Label the new point P'. Open out the paper and join P to P' by a straight line. Check that PP' is at right angles to the mirror line and that P and P' are the same distance from it. Repeat for other points.

Explore reflection using **dynamic geometry software**.
For example:

- Construct a triangle and a line to act as a mirror line. Construct the image of one vertex by drawing a perpendicular to the mirror line and finding a point at an equal distance on the opposite side. Repeat for the other vertices and draw the image triangle.
  Observe the effect of dragging vertices of the original triangle. What happens when the triangle crosses the mirror line?



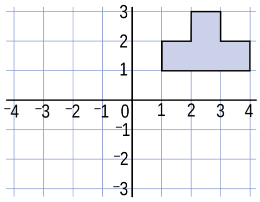
As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

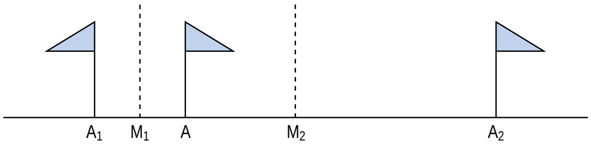
Combinations of two transformations

Transform 2-D shapes by repeated reflections, rotations or translations. Explore the effect of repeated reflections in parallel or perpendicular lines. For example:

- Reflect a shape in one coordinate axis and then the other. For example, reflect the shape below first in the *x*-axis and then in the *y*-axis. What happens? What is the equivalent transformation? Now reflect it first in the *y*-axis and then in the *x*-axis. What happens? What is the equivalent transformation?



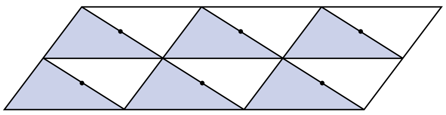
- Investigate reflection in two parallel lines. For example, find and explain the relationship between the lengths AA_2 and M_1M_2.



- Investigate how repeated reflections can be used to generate a tessellation of rectangles.

Explore the effect of repeated rotations, such as half turns about different points. For example:

- Generate a tessellation of scalene triangles (or quadrilaterals) using half-turn rotations about the mid-points of sides.



Explain how the angle properties of a triangle (or quadrilateral) relate to the angles at any vertex of the tessellation.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
plane symmetry, plane of symmetry…
axis of rotation symmetry…

Combinations of transformations

Transform 2-D shapes by combining translations, rotations and reflections, on paper and using **ICT**.

Know that reflections, rotations and translations preserve length and angle, and map objects on to congruent images.

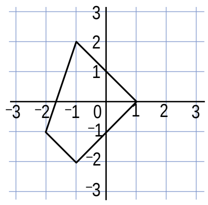
Link to congruence (pages 190–1).

Use mental imagery to consider a combination of transformations and relate the results to symmetry and other properties of the shapes. For example:

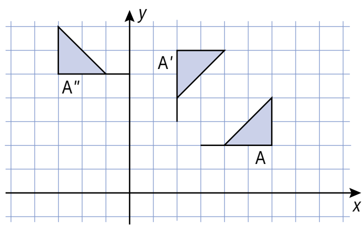
- Say what shape the combined object and image(s) form when:
  a. a right-angled triangle is reflected along its hypotenuse;
  b. a square is rotated three times through a quarter turn about a corner;
  c. a scalene triangle is rotated through 180° about the mid-point of one of its sides.

Working practically when appropriate, solve problems such as:

- Reflect this quadrilateral in the *y*-axis. Then reflect both shapes in the *x*-axis. In the resulting pattern, which lines and which angles are equal in size?



- Flag A is reflected in the line $y = x$ to give A'. A' is then rotated through 90° centre (0, 3) to give A".



Show that A could also be transformed to A" by a combination of a reflection and a translation. Describe other ways of transforming A to A".

Pupils should be taught to:

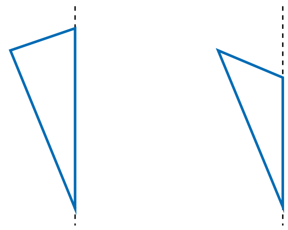
Recognise and visualise transformations and symmetries of 2-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

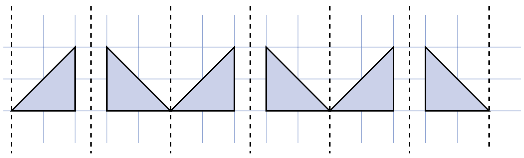
Reflection (continued)

Reflect a shape in a line along one side. For example:

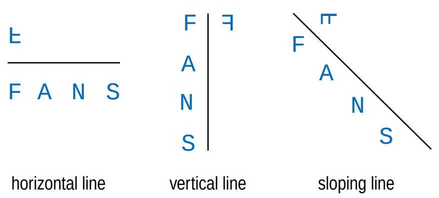
- Reflect each shape in the dotted line.
  What is the name of the resulting quadrilateral?
  Which angles and which sides are equal?
  Explain why.



Reflect a shape in parallel mirrors and describe what is happening. Relate this to frieze patterns created by reflection.

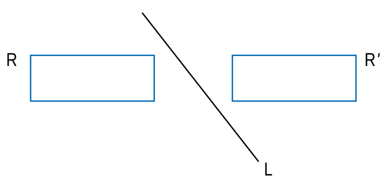


Construct the reflections of shapes in mirror lines placed at different angles relative to the shape. For example:



- Which shapes appear not to have changed after a reflection? What do they have in common?

- In this diagram, explain why rectangle R’ is not the reflection of rectangle R in line L.



As outcomes, Year 8 pupils should, for example:

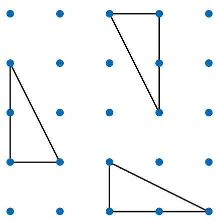
Combinations of transformations (continued)

Understand and demonstrate some general results about repeated transformations. For example:

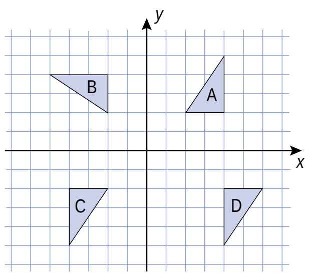
- Reflection in two parallel lines is equivalent to a translation.
- Reflection in two perpendicular lines is equivalent to a half-turn rotation.
- Two rotations about the same centre are equivalent to a single rotation.
- Two translations are equivalent to a single translation.

Explore the effect of combining transformations.

- Draw a 1 by 2 right-angled triangle in different positions and orientations on 5 by 5 spotty paper. Choose one of the triangles to be your original. Describe the transformations from your original to the other triangles drawn. Can any be done in more than one way?



- Triangles A, B, C and D are drawn on a grid.



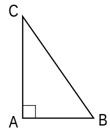
  a. Find a single transformation that will map:
     i. A on to C; ii. C on to D.
  b. Find a combination of two transformations that will map: i. B on to C; ii. C on to D.
  c. Find other examples of combined transformations, such as:
     A to C: with centre (0, 0), rotation of 90°, followed by a further rotation of 90°;
     A to C: reflection in the *y*-axis followed by reflection in the *x*-axis;
     B to C: rotation of 90° centre ($^{-}2$, 2), followed by translation ($^{-}4$, 0);
     C to D: reflection in the *y*-axis followed by reflection in the line $x = 4$;
     C to D: rotation of 270° centre (0, 0), followed by a rotation of 90° centre (4, 4).

Use **ICT**, or plastic or card shapes, to generate tessellations using a combination of reflections, rotations and translations of a simple shape.

As outcomes, Year 9 pupils should, for example:

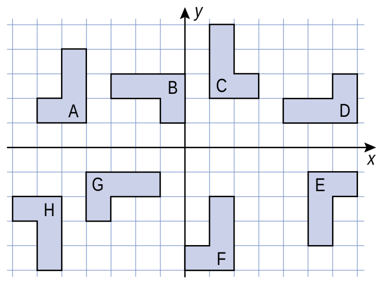
Combinations of transformations (continued)

- ABC is a right-angled triangle. ABC is reflected in the line AB and the image is then reflected in the line CA extended. State, with reasons, what shape is formed by the combined object and images.



- Two transformations are defined as follows:
  – Transformation *A* is a reflection in the *x*-axis.
  – Transformation *C* is a rotation of 90° centre (0, 0).
  Does the order in which these transformations are applied to a given shape matter?

- Some congruent L-shapes are placed on a grid in this formation.



  Describe transformations from shape C to each of the other shapes.

- *Some transformations are defined as follows:*
  – *P is a reflection in the x-axis.*
  – *Q is a reflection in the y-axis.*
  – *R is a rotation of 90° centre (0, 0).*
  – *S is a rotation of 180° centre (0, 0).*
  – *T is a rotation of 270° centre (0, 0).*
  – *I is the identity transformation.*
  Investigate the effect of pairs of transformations and find which ones are commutative.

- *Investigate the effect of a combination of reflections in non-perpendicular intersecting mirror lines, linking to rotation symmetry, the kaleidoscope effect and the natural world.*

*Use **dynamic geometry software** to explore equivalences of combinations of transformations, for example:*

- *to demonstrate that only an even number of reflections can be equivalent to a rotation;*
- *to demonstrate that two half turns about centres C_1 and C_2 are equivalent to a translation in a direction parallel to C_1C_2 and of twice the distance C_1C_2.*

Pupils should be taught to:

Recognise and visualise transformations and symmetries of 2-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

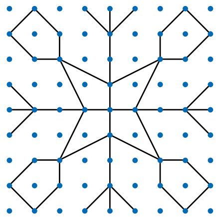
Reflection symmetry

Know that if a line can be found such that one half of a shape reflects to the other, then the shape has reflection symmetry, and the line is called a line of symmetry. Recognise:

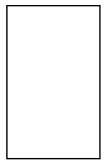
- reflection symmetry in familiar shapes such as an isosceles triangle, a rectangle, a rhombus, a regular hexagon…
- lines of symmetry in 2-D shapes;
- shapes with no lines of symmetry.

For example:

- Identify the lines of symmetry in this pattern.

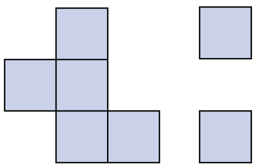


- Imagine that this is how a piece of paper looks after it has been folded twice, each time along a line of symmetry.
  What shape might the original piece of paper have been?
  How many possibilities are there?

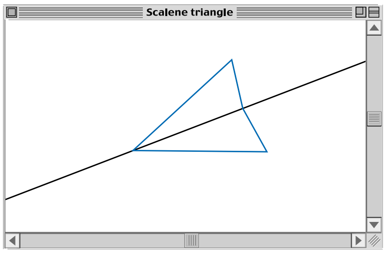


- Combine these shapes to make a shape with reflection symmetry.

  How many different solutions are there? (*Five.*)



- Use **dynamic geometry software** to reflect a triangle in any one of its sides. What shape do the combined object and image form?



  Is this always the case?

See Y456 examples (pages 106–7).

Link to properties of 2-D shapes (pages 186–9).

As outcomes, Year 8 pupils should, for example:

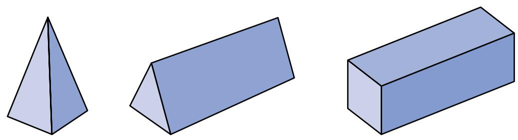
As outcomes, Year 9 pupils should, for example:

Reflection symmetry

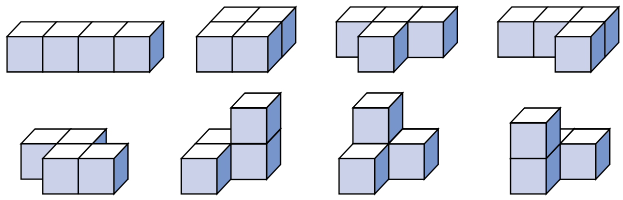
Understand reflection symmetry in three dimensions and identify planes of symmetry.

For example:

- Discuss symmetry in 3-D objects, such as a pair of semi-detached houses...
- Visualise and describe all the planes of symmetry of familiar solids, such as a cube, cuboid, triangular prism, square-based pyramid, regular tetrahedron, regular octahedron.



- Use four cubes to make as many different shapes as possible.



Prove that no other shapes are possible.

For each shape, identify any planes of symmetry. Investigate asymmetrical shapes, looking for any which together form a symmetrical pair.

- *Investigate axes of rotation symmetry for a cuboid.*

Link to properties of 3-D shapes (pages 198–201).

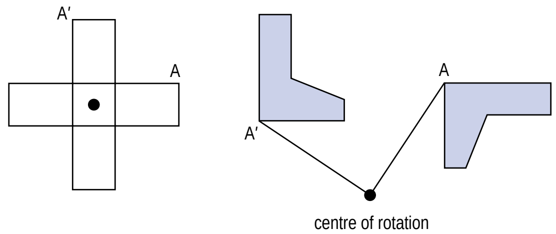
Pupils should be taught to:

Recognise and visualise transformations and symmetries of 2-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

Rotation

Understand rotation in two dimensions as a transformation of a plane in which points (such as A) are mapped to images (A') by turning about a fixed point in the plane, called the centre of rotation.

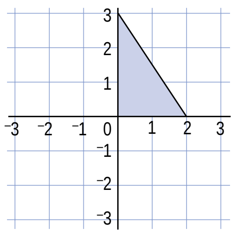


A rotation is specified by a centre of rotation and an (anticlockwise) angle of rotation.

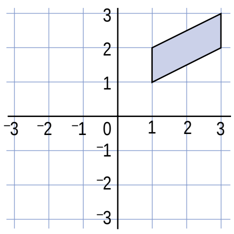
Know that a rotation has these properties:

- The centre of rotation can be inside or outside the shape and its position remains fixed throughout the rotation.
- The inverse of any rotation is either:
  a. an equal rotation about the same point in the opposite direction, or
  b. a rotation about the same point in the same direction, such that the two rotations have a sum of 360°.

Rotate shapes anticlockwise about (0, 0) through right angles and simple fractions of a turn.



Rotate shapes about points other than (0, 0).

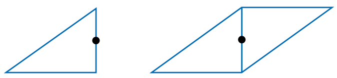


As outcomes, Year 8 pupils should, for example:

Rotation

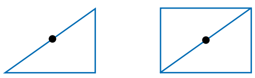
Rotate shapes, and deduce properties of the new shapes formed, using knowledge that the images are congruent to the original and identifying equal angles and equal sides. For example:

- Rotate a right-angled triangle through 180° about the mid-point of its shortest side.



Name the shape formed by the object and image.
Identify the equal angles and equal sides.
Explain why they are equal.

What happens when you rotate the triangle about the mid-point of its longest side?



As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Recognise and visualise transformations and symmetries of 2-D shapes (continued)

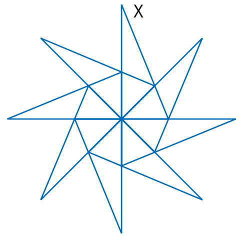
As outcomes, Year 7 pupils should, for example:

Rotation symmetry

Know that:
- A 2-D shape has rotation symmetry of order n when n is the largest positive integer for which a rotation of 360° ÷ n produces an identical looking shape in the same position.
- The order of rotation symmetry is the number of ways the shape will map on to itself in a rotation of 360°.

For example:

- This shape has rotation symmetry of order 8 because it maps on to itself in eight distinct positions under rotations of 45° about the centre point.

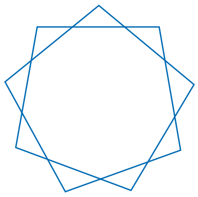


Solve problems such as:

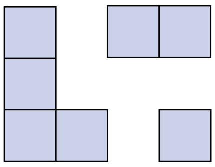
- Prove that the central shape in the above diagram is a regular octagon.
  What shape is traced by point X as it moves through one complete revolution?

- Recognise the rotation symmetry of familiar shapes, such as parallelograms and regular polygons.

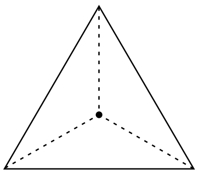
- Use **Logo** or other **ICT resource** to produce shapes with a specified order of rotation symmetry, e.g. 9.



- Combine these shapes to make a single shape with rotation symmetry of order 2.



- Identify the centre of rotation in a shape with rotation symmetry, such as an equilateral triangle.
  Justify the choice.



Link to properties of 2-D shapes (pages 186–9).

As outcomes, Year 8 pupils should, for example:

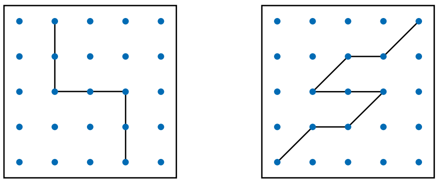
As outcomes, Year 9 pupils should, for example:

Reflection symmetry and rotation symmetry

Recognise all the symmetries of 2-D shapes.

For example:

- Make polygons on a 3 by 3 pinboard. Explore:
  - the maximum number of sides;
  - whether any of the polygons are regular;
  - symmetry properties of the polygons…
- Recognise the rotation symmetry in congruent divisions of a 5 by 5 pinboard.



Identify and describe the reflection and rotation symmetries of:
- regular polygons (including equilateral triangles and squares);
- isosceles triangles;
- parallelograms, rhombuses, rectangles, trapeziums and kites.

Devise a tree diagram to sort quadrilaterals, based on questions relating to their symmetries. Compare and evaluate different solutions.

Relate symmetries of triangles and quadrilaterals to their side, angle and diagonal properties.
For example:

- An isosceles triangle has reflection symmetry. Use this to confirm known properties, such as:
  - The line of symmetry passes through the vertex which is the intersection of the two equal sides, and is the perpendicular bisector of the third side.
  - The base angles on the unequal side are equal.
- A parallelogram has rotation symmetry of order 2, and the centre of rotation is the intersection of the diagonals.
  Use this to confirm known properties, such as:
  - The diagonals bisect each other.
  - Opposite sides and opposite angles are equal.
  - The angles between a pair of opposite sides and a diagonal are equal.

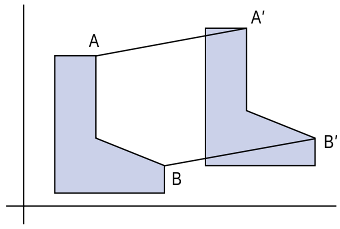
Link to properties of 2-D shapes (pages 186–91).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Recognise and visualise transformations and symmetries of 2-D shapes (continued)

Translation

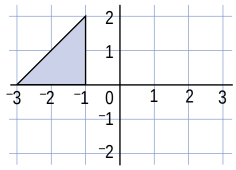
Understand translation as a transformation of a plane in which points (such as A and B) are mapped on to images (A' and B') by moving a specified distance in a specified direction.



Know that when describing a translation, it is essential to state either the direction and distance or, with reference to a coordinate grid, the moves parallel to the x-axis and parallel to the y-axis.

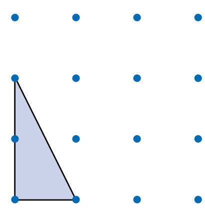
Know that a translation has these properties:
- The orientations of the original and the image are the same.
- The inverse of any translation is an equal move in the opposite direction.

Translate shapes on a coordinate grid, e.g. 4 units to the right, 2 units down, then 3 units to the left. Determine which two instructions are equivalent to the three used.



Investigate translations. For example:

- What are the possible translations of a 1 by 2 right-angled triangle on 3 by 3 pinboard?
  What about a 4 by 4 pinboard, then a 5 by 5 pinboard?



- Examine a tessellating pattern (e.g. of equilateral triangles, squares or regular hexagons).
  Start from one particular shape in the middle.
  Identify translations of that shape, and the direction of the translation.

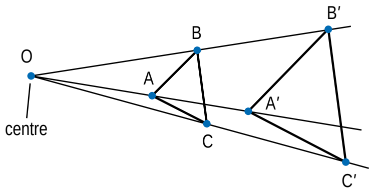
See Y456 examples (pages 106–7).

As outcomes, Year 8 pupils should, for example:

Enlargement

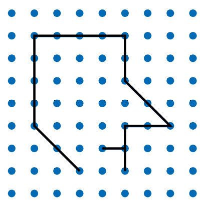
Use, read and write, spelling correctly:
enlarge, enlargement, centre of enlargement...
scale, scale factor, ratio...
scale drawing, map, plan...

Understand enlargement as a transformation of a plane in which points (such as A, B and C) are mapped on to images (A', B' and C') by multiplying their distances from a fixed centre of enlargement by the same scale factor. In this example, triangle ABC maps to A'B'C':



$$\text{scale factor} = \frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

- Draw a simple shape on a 1 cm spotty grid, e.g. a 'standard angular person'.



Choosing a suitable centre, enlarge the shape by different positive scale factors, such as ×2, ×3, ×4, (double person, treble person, quadruple person). Construct a table of measurements.

Scale factor	×1	×2	×3	×4
Width of head (cm)	4			
Width of neck (cm)	2			
Full height of head (cm)	6			
Width of mouth (cm)	1			
Diagonal length of nose (cm)	2.8			

Check that the ratio of corresponding linear measurements is always equal to the scale factor.

Experiment with different centres.

Begin to understand the property that the ratios of corresponding lengths in the image and in the object are equal to the scale factor, and to recognise this as a constant proportion:

$$\text{scale factor} = \frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$$

As outcomes, Year 9 pupils should, for example:

Enlargement

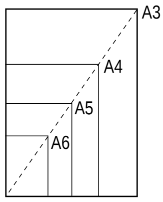
Use vocabulary from previous year and extend to:
similar...

Understand and use the definition of enlargement from a centre. Recognise that:
- The object and its image are similar.
- The ratio of any two corresponding line segments is equal to the scale factor.

- *Extend the 'standard angular person' activity to enlargements by a fractional scale factor, such as ½ or ¼.*

- Investigate the proportions of metric paper sizes, A6 to A1. For example, start with a sheet of A3 paper and, with successive folds, produce A4, A5 and A6.

  Demonstrate practically that the different sizes of paper can be aligned, corner to corner, with a centre of enlargement.



  Confirm by measurement and calculation that the scale factor of enlargement is approximately 0.7.

  Follow an explanation that, if the metric paper has dimensions h and w, then $h : w = w : h/2$. Deduce that $h = \sqrt{2}\,w$.

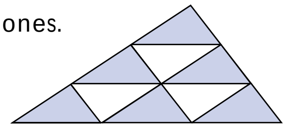
- Compare a simple shape with enlargements *and reductions* of it made on a photocopier. Estimate the scale factors of the enlargements as accurately as possible.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Recognise and visualise transformations and symmetries of 2-D shapes (continued)	

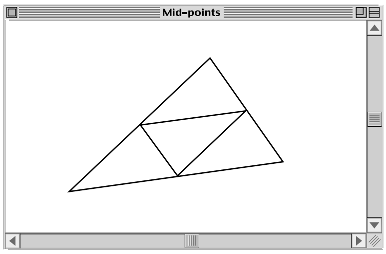
As outcomes, Year 8 pupils should, for example:

Explore some practical activities leading to consideration of enlargement. For example:

- Build triangles into bigger ones.



- Use **dynamic geometry software** to draw a triangle. Join the mid-points of the sides. Observe the effect as the vertices of the original triangle are dragged. Describe the resulting triangles.



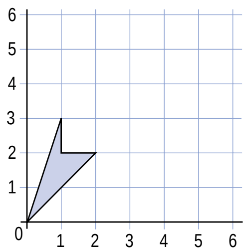
From practical work, appreciate that an enlargement has these properties:
- An enlargement preserves angles but not lengths.
- The centre of enlargement, which can be anywhere inside or outside the figure, is the only point that does not change its position after the enlargement.

Discuss common examples of enlargement, such as photographs, images projected from slide or film, images from binoculars, telescopes or microscopes.

Know that when describing an enlargement the centre of enlargement and the scale factor must be stated.

Enlarge 2-D shapes, given a centre of enlargement and a positive whole-number scale factor. For example:

- Draw a simple shape on a coordinate grid. Take the origin as the centre of enlargement. Enlarge the shape by a whole-number scale factor.



Relate the coordinates of the enlargement to those of the original.

Link to ratio and proportion (pages 78–81).

As outcomes, Year 9 pupils should, for example:

Describe and classify some common examples of reductions, e.g. maps, scale drawings and models (scale factors less than 1).

Recognise how enlargement by a scale factor relates to multiplication:
- *Enlargement with scale factor k relates to multiplication by k.*
- *The inverse transformation has scale factor $\frac{1}{k}$ and relates to multiplication by $\frac{1}{k}$.*
- *The terms 'multiplication' and 'enlargement' are still used, even when the multiplier or scale factor is less than 1.*
- *Two successive enlargements with scale factors k_1 and k_2 are equivalent to a single enlargement with scale factor k_1k_2.*

Understand that enlargements meet the necessary conditions for two shapes to be mathematically similar, i.e. corresponding angles are equal and corresponding sides are in the same ratio.

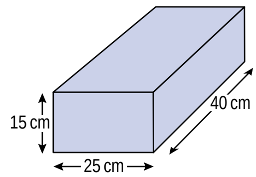
Understand the implications of enlargement for area and volume.
Know that if a shape is enlarged by a scale factor k, then its area (or surface area) is enlarged by scale factor k^2 and its volume by scale factor k^3.

For example:

- *Find the area covered by the standard angular person's head (see page 213), and the areas covered by enlargements of it. Tabulate results and compare scale factors for length and for area. Confirm that the scale factor for area is equal to the square of the scale factor for length.*

- *Start with a unit cube or a simple cuboid, enlarge it by a chosen scale factor and compare with the scale factors for surface area and volume.*

- *Find the surface area and volume of this cuboid.*



  Find the surface area and volume after you have:
  a. doubled its length;
  b. doubled both its length and its width;
  c. doubled each of its length, width and height.

  What are the relationships between the original and the enlarged surface area and volume?

Appreciate some of the practical implications of enlargement, e.g. why a giant would tend to overheat and find standing upright rather painful.

Link to ratio and proportion (pages 78–81), and similarity (pages 192–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Use and interpret maps and scale drawings	

As outcomes, Year 8 pupils should, for example:

Use scales and make simple scale drawings.

For example:

- Design a layout for a bedroom, drawing the room to scale and using cut-outs to represent furniture.
- Estimate from a photograph the height of a tall tree or building by comparison with a person standing alongside it.
- On a sunny day, estimate the height of a telegraph pole or tall tree by using shadows.
- Find the scale of each of these:
  a. the plan of a room with 2 cm representing 1 m;
  b. the plan of the school field with 1 inch representing 50 yards;
  c. a map of the area surrounding the school, with 4 cm representing 1 km.

Link to measuring lengths and conversion of one unit of measurement to another (pages 228–31), and to ratio and proportion (pages 78–81).

As outcomes, Year 9 pupils should, for example:

Use and interpret maps and scale drawings in geography, design and technology and other subjects as well as in mathematics.

Understand different ways in which the scale of a map can be represented and convert between them, e.g. 1 : 50 000 or 2 cm to 1 km.

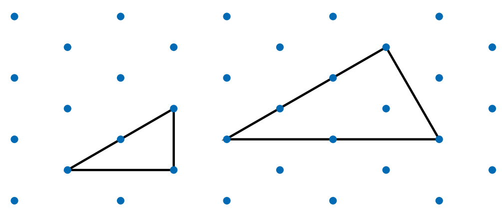
Understand that in a scale drawing:
- linear dimensions remain in proportion, e.g. actual length of object : actual width of object = scaled length : scaled width;
- angles remain the same, e.g. the slope of the floor in a cross-sectional drawing of a swimming pool will be the same as it is in reality.

Measure from a real map or scale drawing.
For example:
- Use the scale of a map to convert a measured map distance to an actual distance ‘on the ground’.
- Measure dimensions in a scale drawing and convert them into actual dimensions.

Understand that maps, plans and scale drawings are examples of enlargement by a fractional scale factor.

Understand the implications of enlargement for the area of a scale drawing. For example:

- *On a map of scale 1 : 25 000, a given distance is represented by a line **twice** the length of the corresponding line on a 1 : 50 000 map. Show that this requires a sheet **four** times the area to cover the same ground.*
- *Show that these two triangles are similar.*
  Find the ratio of the areas of the two triangles.



Link to measuring lengths and conversion of one unit of measurement to another (pages 228–31), ratio and proportion (pages 78–81), and area and volume (pages 234–41).