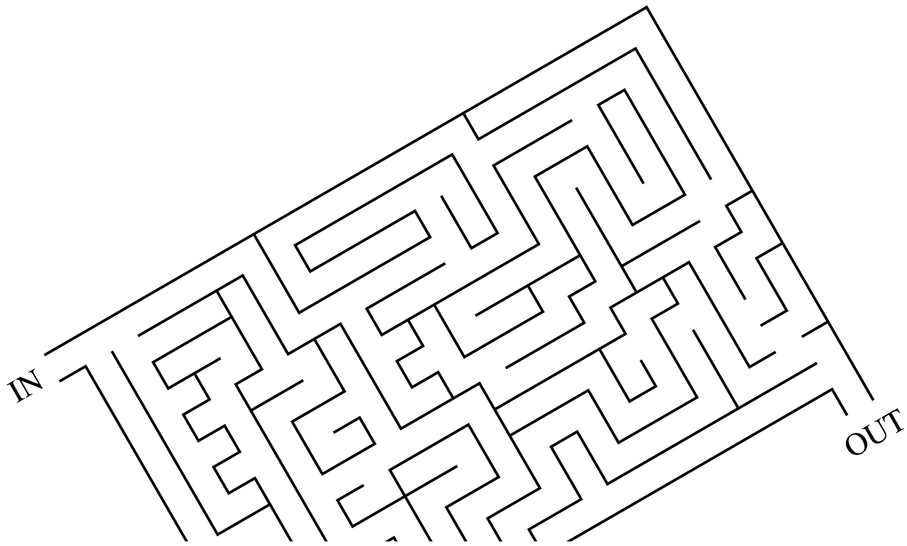
Getting Started

Mathematical Activitites for Key Stage 3 and 4

Jim Smith



The Mathematical Association

Getting Started – Mathematical Activities for Key Stage 3 and 4

'The teachers job is to organise and provide the sorts of experience which enable pupils to construct and develop their own understanding of mathematics, rather than simply communicate the ways in which they themselves understand the subject.' NCC Non-statutory Guidance (1989, p2.2)

Introduction:

Getting Started is a collection of ideas for presenting mathematics in an interactive manner with the aim of creating pupil involvement and sustaining it through active learning tasks. Whilst originally written for student teachers, the activities are offered as starting points which could be developed to suit any individual teacher's approach and adapted to suit the needs of particular classes.

The activities have been tried and tested in the classroom by myself, student teachers, new teachers and experienced teachers in a range of schools. In many cases the comments of these teachers have led to modifications to the text, and in some cases the comments have been added in the form of a commentary.

To get an unfamiliar activity started in the classroom is not easy and so considerable attention has been paid to giving a suggested presentation of each activity. These are not intended to be prescriptive, but to help the experienced teacher to visualise a possible approach and to offer the new teacher an approach which has worked for others.

The 'pupil activities' are initiated by the teacher presentation, but need to be sustained through continued teacher support and involvement. Some variations of the activities are suggested, as well as ideas for ensuring progression onto more advanced work.

Acknowledgements:

I am grateful to various colleagues who suggested activities, commented on early drafts and tried out materials, in particular staff in the mathematics departments of Bradfield, Birley, Chaucer and Waltheof Schools in the Sheffield area. I would particularly like to acknowledge the contribution made by Denise Smith in commenting on activities, testing some in practice and assisting with the production of final proofs. Any remaining errors are my own.

Correspondence:

Correspondence on the activities and suggestions for further resources are welcomed.

Jim Smith, Sheffield Hallam University Mathematics Education Centre,

25, Broomgrove Road, Sheffield, S10 2NA.

Tel. 0114 253 2349, Fax. 01142 253 2339, Messages. 0114 253 2343

E-mail D.N.Smith@ shu.ac.uk

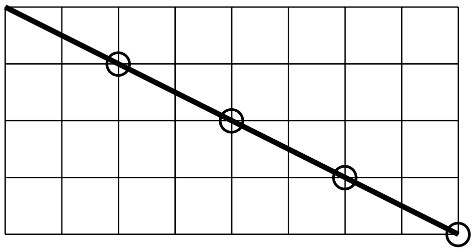
Contents

Contents (continued)

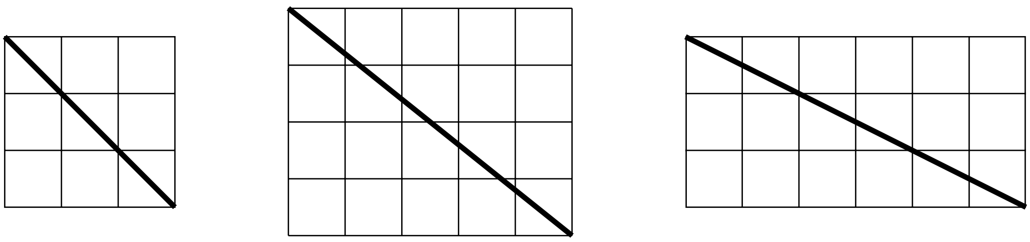
Diagonals of Rectangles

Purpose:
To introduce the concept of the Highest Common Factor.

Presentation:
Invite pupils to look at a diagram of a rectangular grid, say 8 by 4, whilst the teacher draws in just one diagonal line as shown on the right.



Count up the number of times that the diagonal line simultaneously crosses the horizontal and vertical grid lines, except for the first time. In the above case, the answer would be the points circled, i.e. 4. To check for understanding, try one or two more examples, like these:



The counts here being 3, 1 and 3 respectively.

Pupil Activity:
To produce some more results and find a way of predicting the count for a grid of any size. The diagrams need to be accurate and pupils should use sharp pencils.

Variations:
1) instead of counting intersections with both grid lines simultaneously, count the number of intersections with any grid line. For the three diagrams above, this would yield results of 4, 9 and 7 respectively.

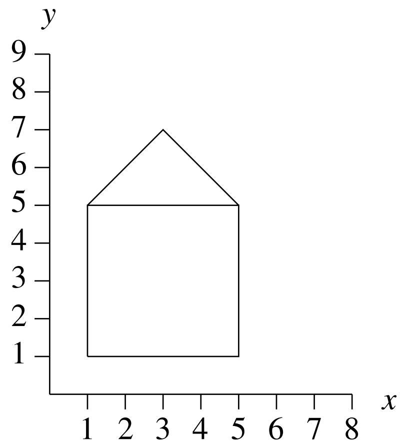
2) count the number of squares crossed. For the three diagrams above, this would yield results of 3, 8 and 6 respectively.

Progression:
Pupils need to know that the function that they find which works for the first investigation is called Highest Common Factor. Some practice in finding the HCF would seem appropriate, particularly extending the concept to cover HCF of any size set of natural numbers.

Using scale factors

Purpose:
To consider the effect upon area and volume of enlarging a shape by various scale factors.

Presentation:
Pupils could be asked to draw a simple shape by connecting the coordinates of each of the vertices; for example (1, 5) joined to (3, 7) to (5, 5) to (1, 5) again, then to (1, 1) and (5, 1) and back to (5, 5). The shape will need to be drawn on large enough axes for the enlargements to take place.



Pupil Activity 1:
To investigate the effect on the shape of doubling the coordinates and drawing the ‘new’ shape. Is it the same shape? What is it that changes and what does not change?

To try some simple shapes of their own and follow the same procedure and ask themselves the same questions.

What happens if the coordinates are trebled? multiplied by four? five? n?
This can produce some attractive display work for the classroom.

Pupil Activity 2:
This time pupils can be asked to focus specifically on the changes in area; perhaps completing a table of results. The aim here is to find a connection between the multiplier and the effect on the area.

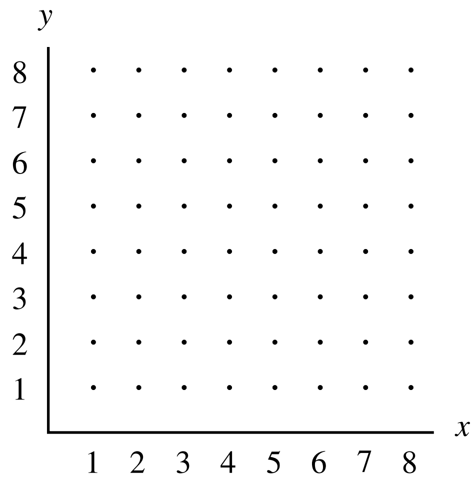
Variations:
What happens if you divide the coordinates by a fixed amount? add on an amount? multiply by two then add on four? etc. etc.

Progression:
A similar activity can be devised using interlocking plastic cubes to explore the effect of enlargement upon volume and surface area.

Four in a Row

Purpose:
To practice coordinates and develop pupils' understanding of graphical relationships.

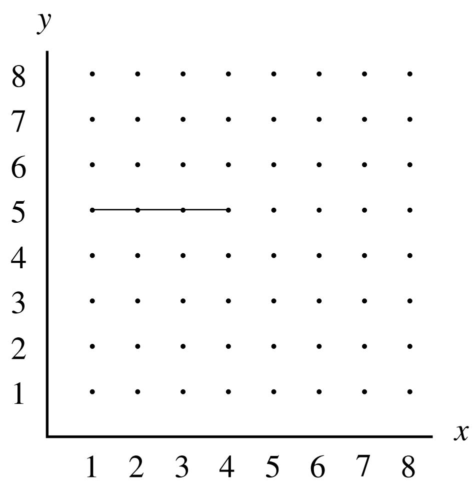
Presentation:
As the activity is a game, a simple and effective way of getting started is to play a game or two with the whole class. On the board you will need a coordinate grid as shown on the right.

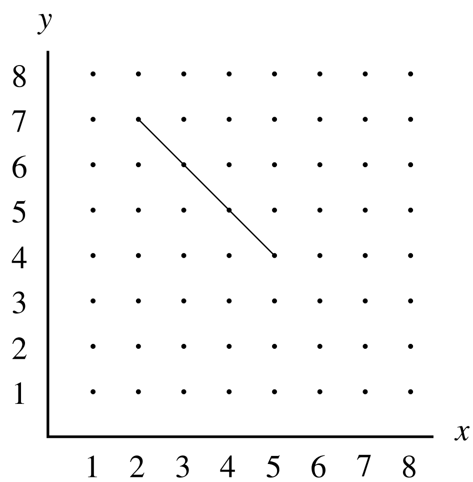


The teacher and the class take it in turns to place a O or X on the grid points. Pupils should use coordinates to describe where they wish to place their turn. The winner is the first to get four points in a line.

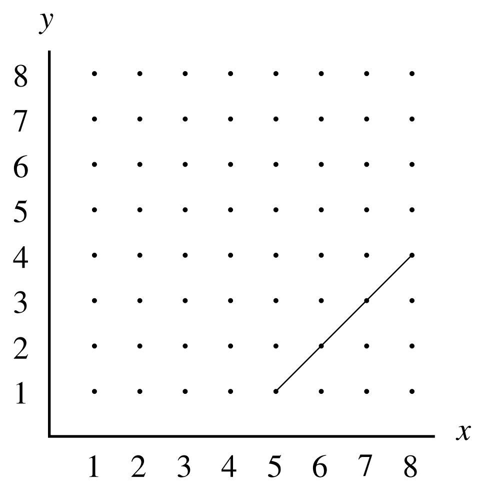
Pupil activity 1:
Pupils to work in pairs on this for a while, perhaps on pre-printed grids, to save time.

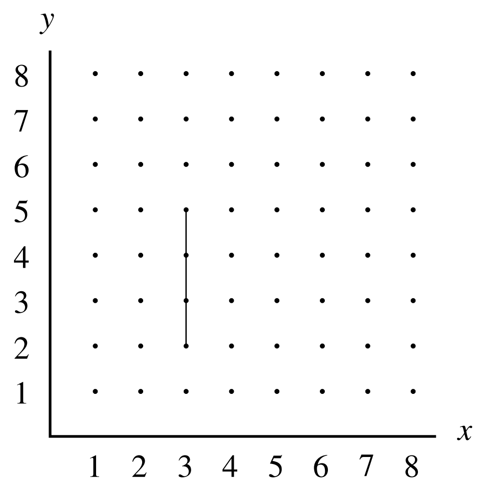
Teacher Intervention:
What is special about the coordinates of the winning lines of these





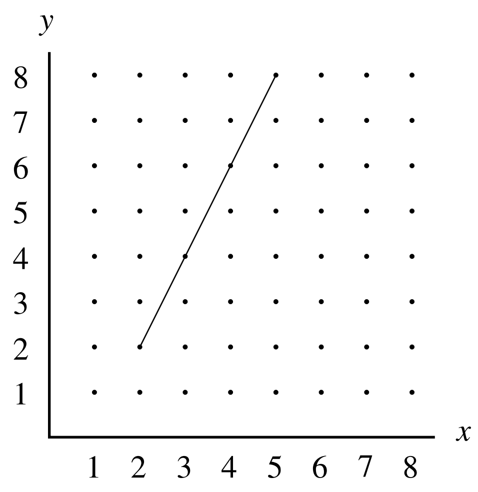
The intention here is to introduce eventually equations like $y = 5$, $x + y = 9$ and $y = x - 4$.

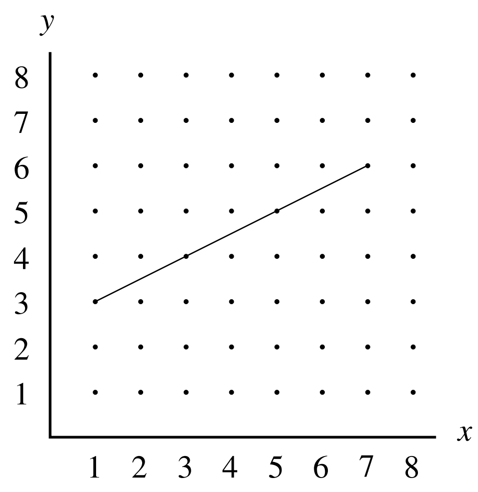




Pupil Activity 2:
Pupils are to continue their games but the additional rule is introduced that 'You are not allowed to win unless you can correctly state the equation of the winning line'. The teacher could act as referee.

Variations:
Continue until the grid is full and the winner is the player with most winning lines. Investigate how many winning lines there could be on an 8×8 grid and give their equations. What about winning lines like these …?





Progression:
Onto games played in all four quadrants.

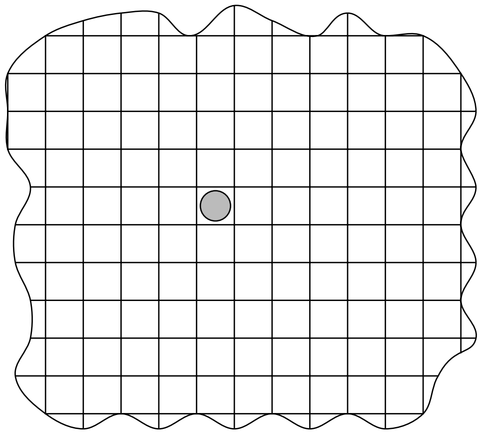
Teacher Comments:
'Our department have used this type of activity before, but this version has extended the idea into algebra for us.'

The Game of Death

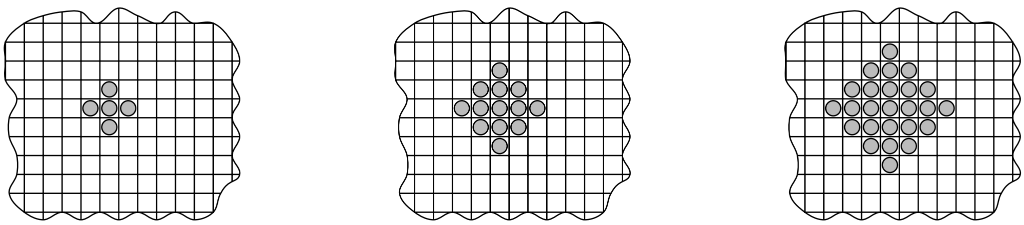
Purpose:
An activity which leads to quadratic functions.

Presentation:

Pupils are presented with a grid representing part of an enormous store of apples and are asked to imagine that there is one bad apple in the store.



Each day the rot spreads to adjacent cells in the grid, so that the following sequence of pictures illustrates the store over the next three day period.



Pupil Activity:
Pupils could be asked to predict the growth of the rot over the next 10 days or so and to find a functional relationship between the total number of rotten apples and the number of days. How many days pass before a million apples have rotted?

Variations:
Pupils to explore what occurs with different numbers and locations of rotten apples at the start. For example, starting with one apple leads to a particular formula, starting with two adjacent leads to another formula, what about starting with *n* adjacent apples?
What would happen if the apples were adjacent diagonally to begin with?

Progression:
Questions like 'How many days pass before a million apples have rotted?' can lead into solving quadratic equations and inequalities.

Grids and Overlays

Purpose:
To encourage pupils to investigate number patterns, make and test predictions.

Presentation:
This is best presented on an OHP transparency of a 100 square, as illustrated, together with a separate transparency of a T shape.

1	2	3	4	5	6	7	8	9	10
11	12	13	**14**	**15**	**16**	17	18	19	20
21	22	23	24	**25**	26	27	28	29	30
31	32	33	34	**35**	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Place the T onto the hundred grid and ask pupils to describe anything that they notice about the numbers in the T shape.

Pupils generally notice all kinds of things. In particular, if nobody else mentions it, ask them to notice the total.

Move the T shape somewhere else on the grid and repeat the above discussion, noting in particular which things change and which do not change.

In particular, what has happened to the total?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	**56**	**57**	**58**	59	60
61	62	63	64	65	66	**67**	68	69	70
71	72	73	74	75	76	**77**	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Focussing from now only on the total....

What happens to the total if I move the T one square right? left? down? up?

What happens to the total if I move the T one square right and one square down?

Try some other movements.

Pupil Activity 1:
Pupils can work on pre-printed 100 squares and use tracing paper for their shape.

Ask pupils to make up their own shape (like the T, but not the same T), and to investigate what happens to the total in their shape when it is moved around the grid.

Can their total ever be 250?

Pupil Activity 2:
To repeat Activity 1 for a different shape, but to try to predict their results before they confirm or refute them.

Variations:
Different shapes can be used. The grid can be numbered differently. Different transformations can be used, e.g. we can consider the effect of reflections, rotations and enlargements.

Progression:
Algebra can be introduced, for example

x	$x + 1$	$x + 2$
	$x + 11$	
	$x + 12$	

This has a total of $5x + 35$, and to find out if the total can ever be 250 we can solve $5x + 35 = 250$ to obtain $5x = 215$ so $x = 43$. This works out on the grid, as the T shape will not go over the edge. Incidentally, what is special about values of x which do not force the T shape over the edge?

Teacher Comments:
‘This has proved to be an activity which pupils enjoy, probably because there are many opportunities for them to make up their own questions and to follow their own lines of inquiry.’