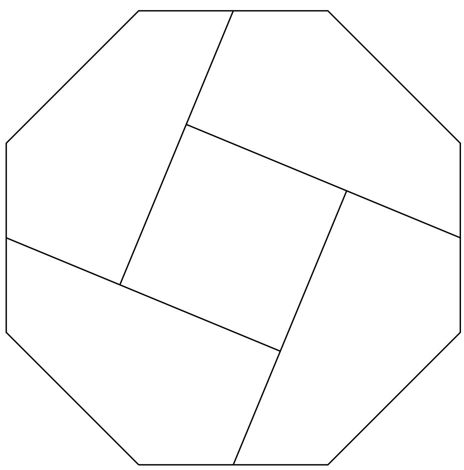
THE SCOTTISH MATHEMATICAL COUNCIL

journal 29



JOURNAL 29

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Views expressed in the articles in this Journal are those of the authors and do not necessarily represent the official view of the Scottish Mathematical Council.

The Council would welcome submitted articles and correspondence. Those interested should contact the Secretary, the Journal Editor or any other member of the Council.

The Journal is an annual publication which contains articles of general interest to those interested in Mathematical Education, reports of conferences and Mathematical Competitions, news of forthcoming events and other items of national interest. It is available free to those affiliated to the Scottish Mathematical Council and on annual subscription of £10 to others.

The Scottish Mathematical Council 1999 – 2000

SMC Web page address is http://www-maths.mcs.st-and.ac.uk/smc/

From the Editor

Once more I have the opportunity to thank all those who have given up precious time to make their contributions to this journal. This year we have only three articles based on talks and presentations made at the 1999 Stirling Conference but these are accompanied by a variety of others from able authors with an involvement and interest in mathematics education.

Despite the waiver on the contents page, this year we open with two articles which do indeed represent the views of the SMC. The first expresses heartfelt congratulations to our chairman, Adam McBride. on being awarded the OBE and the second is our Position Paper on the future of Mathematics education in Scotland.

These are followed by reports on Mathematical Challenge, Mathematics Masterclasses and the 1999 Mathematical Olympiad and UKMT and its competitions. If we had thought sooner perhaps we could have included an article on Enterprising Mathematics which by all accounts was again a tremendous success this year. Maybe next year.....

There are three articles relating to technology in Mathematics teaching, one on the fact that girls now appear to be outperforming boys in Mathematics (as well as in so many other subject areas), two on basic numeracy and its teaching, one on a school's Praise system for improving motivation, discipline and performance and two others which could safely be described as mathematical!

I trust you will enjoy reading the journal as much as I have and hope you agree that the quality of previous years has been maintained.

Any contributions or suggestions for future journals will be gratefully received, ideally by early September 2000.

Allan Duncan
Lecturer in Mathematics Education, Northern College, Aberdeen

Copying of Material from the Journal

The Scottish Mathematical Council

The main objects of the Council are to foster and improve mathematical education at all levels, and to encourage the advancement and application of mathematics throughout Scotland. For this purpose the Council collaborates with educational, technological and industrial organisations and undertakes functions such as the following:

1. The promotion and coordination of conferences, special courses and instruction and retraining schemes.
2. The consideration of factors relating to the supply and deployment of trained mathematicians with special reference to the supply of teachers.
3. The giving of advice on curricula, syllabuses, examinations and other subjects of mathematical interest.
4. The collection and circulation of information and the publication of reports, articles and other material of pedagogic interest.
5. The setting up of committees to consider and report on special problems and subjects.
6. The trusteeship of trusts established for the purposes associated with the objects of the Council.
7. The running of the problem solving competition 'Mathematical Challenge' for primary and secondary school pupils.

The annual journal is the main instrument for reporting on the above activities. It is published annually and sent to all secondary schools in Scotland and to many interested bodies in the UK and abroad.
Details of the Council's School Competitions appear later in the Journal.

The Council would like to thank the following authorities for their sponsorship and other support:

Aberdeen City Council
Aberdeenshire Council
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Argyll & Bute Council
Clackmannanshire Council
Dumfries & Galloway Council
Dundee City Council
East Ayrshire Council
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City of Edinburgh Council
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Highland Council
Inverclyde Council
Midlothian Council
Moray Council
North Ayrshire Council
North Lanarkshire Council
Orkney Islands Council
Perth & Kinross Council
Renfrewshire Council
Scottish Borders Council
Shetland Islands Council
South Ayrshire Council
South Lanarkshire Council
Stirling Council
West Dunbartonshire Council
West Lothian Council
Western Isles Council

(on behalf of their schools)

Thanks are also due to all the Scottish Universities, Colleges, Teachers, Advisers and HMIs who contribute to the running of the Council's activities and so make these all the more successful.

The Council would like to thank the following organisations for their sponsorship and other support:

BAeSEMA
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FALKIRK COLLEGE
GLASGOW MATHEMATICAL ASSOCIATION
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SCOTTISH AMICABLE
SCOTTISH INTERNATIONAL EDUCATION TRUST
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TEXAS INSTRUMENTS
UNIVERSITY OF ABERDEEN
UNIVERSITY OF STIRLING
ZENECA

The Council is grateful to the following affiliated bodies for their support:

Affiliated Institutions

Aberdeen Mathematical Association
Dundee Mathematical Society
Edinburgh Mathematical Society
Faculty of Actuaries
Glasgow Mathematical Association

Affiliated Schools

Albyn School for Girls, Aberdeen
Craigholme School, Glasgow
Dollar Academy
Fettes College, Edinburgh
George Watson's College, Edinburgh
The Glasgow Academy
High School of Dundee
The High School of Glasgow
Hutchesons' Grammar School, Glasgow
Keil School, Dumbarton
Kelvinside Academy
Laurel Park School, Glasgow
Loretto School, Musselburgh
Morrisons Academy, Crieff
Robert Gordon's College, Aberdeen
St Aloysius' College, Glasgow
St Denis and Cranley School, Edinburgh
St George's School for Girls, Edinburgh
St Leonard's School, St Andrews
St Margaret's School for Girls, Aberdeen
St Margaret's School for Girls, Edinburgh
St Serf’s School, Edinburgh

Jordanhill School, Glasgow

Institutions, societies and organisations that are involved in mathematical teaching, research or applications may seek affiliation to the Council. The Council shall have power to grant or refuse requests for affiliation. If affiliation is approved by the Council, the institution concerned shall pay an agreed annual affiliation subscription to the Council.

Adam Clark McBride OBE

Anyone in the Scottish mathematical community (and also many beyond) who studied the Birthday Honours List on Saturday June 12th 1999 would have been delighted to see the name of Adam McBride in the category *Officer of the Order of the British Empire*. It was, in the opinion of many, a long overdue Honour. Adam has always worked and continues to work tirelessly for mathematics at all levels. Some of the activities which no doubt influenced the decision to honour him are that:–

- Within schools, he is in demand as a visiting speaker for senior pupils. He has been happy to travel to distant venues. He has spoken in many schools and is always a passionate advocate of the subject.
- As well as the semi-formal lecture type situation outlined above, he has willingly given up time to spend weekends with aspiring mathematicians. He has assisted at a residential event for senior pupils organised by Lothian Region and he has also been involved in similar activity weekends organised by the Highland Region. There may well have been others for other groups.
- He has given freely of his time to inspire teachers. For many years, he has been involved with a Scottish sub-committee of the Mathematical Association. He is, of course, highly active. The group organised several area conferences on the theme of 5 to 14 mathematics. Adam was always the closing speaker to send the customers (many of whom were Primary School teachers) away totally inspired. Also within the Mathematical Association, he is a strong supporter of its main publication, *The Mathematical Gazette*, and this support is shown in his activities as a writer, referee and book reviewer.
- Masterclasses are another activity which he has supported. These are for pupils aged about 14 and have been held on Saturday mornings in Edinburgh and in Glasgow. Each year, Adam has been a speaker at one or two of these sessions.
- Adam has been a strong supporter of the Scottish examination system. A few years ago, he was the convener of the Mathematics subject panel, a body which has a major influence in syllabus review and implementation and other related matters. He has always taken a particular interest in the final year examinations (Certificate of Sixth Year Studies) and has been called upon as a moderator on a very frequent basis.
- Of course, another school orientated body which has benefitted from Adam's enthusiasm is the Scottish Mathematics Council. He was an 'ordinary' member several years ago. Such involvements are time limited but, after a break, Adam was invited back to be the Chairman. As chairman, he does a tremendous job of inspiring the whole committee. Over many years, Adam has been a speaker in both smaller groups as well as in plenary sessions at the SMC conference. He has been a frequent contributor to the Journal over many years.
- Beyond Scotland, Adam is the vice-chairman of the UK Mathematics Trust. Within that body, he is the chairman of the Olympiad subtrust and has lead the UK team to several recent Olympiads. He is called on by both the London and Edinburgh Mathematical Societies and is active in both.

So, because of all these things and probably many others, Adam went to Buckingham Palace on Friday 12th November. The investiture that day was conducted by HRH Prince Charles, no doubt with all due pageantry. It would have been a colourful occasion and it will be good to see the photos of a kilted McBride receiving what all mathematicians feel to be his just desserts.

Congratulations Adam from all of us.

The SCOTTISH MATHEMATICAL COUNCIL

The Future of Mathematics Education in Scotland

A Position Paper on Raising Attainment in Schools

Introduction

The Scottish Mathematical Council (SMC) was set up in 1967. According to its Constitution

> 'The main objects of the Council shall be to foster and improve mathematical education at all levels and to encourage advancement and application of mathematics throughout Scotland.'

Further, among the functions of the Council listed in the Constitution is

> 'the giving of advice on curricula, syllabuses, examinations and other subjects of mathematical interest'.

The SMC has 16 members drawn from primary and secondary education, further and higher education, teacher education institutions, advisers, HMI and the world of industry and commerce. The SMC is the only mathematical body based in Scotland with such a wide membership. The Council is therefore particularly well qualified to survey the scene in mathematical education in Scotland at the dawn of the new Scottish Parliament and the new Millennium.

Discussion

1. Shortcomings in the present system have received wide publicity and have generated lively debate in the media during recent months. The poor performance of Scotland in the Third International Mathematics and Science Survey (TIMSS) has attracted much attention. Another worry is the apparent decline in mental ability in mathematics between P7 and S2 as reported in the 1997 Assessment of Achievement Programme (AAP).

2. The last few years have seen a large number of initiatives in all aspects of education, and particularly in mathematics. Indeed there have been so many that teachers could be said to be suffering from 'initiative fatigue'.

3. The HMI report 'Improving Mathematics Education: 5-14' makes various recommendations which have commanded wide support. However, it is felt that a vacuum exists and that the recommendations need to be broadened out considerably to tell teachers how to achieve the desired results.

4. To implement these initiatives teachers need considerable support and adequate resources. It has been suggested that at present there is

   'too much pushing, not enough leading'

   from the top.

5. There is no direct analogue in Scotland of the National Numeracy Project south of the border. Nevertheless, good work is being done at certain levels. In particular, the various Early Intervention Schemes seem to be producing encouraging results.

RECOMMENDATIONS

6. **There should be no fresh initiatives for the next few years.**

7. **The implementation of existing initiatives needs to be co-ordinated on a national basis with far greater levels of support and resources.**
This should be effected by means of a full-time Task Force of at least 6 people, specifically appointed, with national responsibilities.

8. **Led by a National Co-ordinator, the Task Force should consist of specialists who, between them, cover the whole age range from nursery to upper secondary.**

9. **The Task Force should be responsible for all necessary in-service training. There must be NO CASCADING. It is not enough to train trainers. Every mathematics teacher should receive training DIRECTLY.**

Further Discussion

10. These recommendations would ensure maximum benefit from existing initiatives. In addition, the creation of a Task Force would facilitate
the identification and dissemination of good practice.
The implications for raising of attainment among pupils are obvious.

11. To implement the required training, use could be made of PAT hours or closure days. In this case, at least one extra day should be set aside for the next two years. An alternative would be to make money available to pay teachers to attend training courses on Saturday mornings.

12. As well as conducting training, the Task Force should be responsible for coordinating centrally produced materials. The model used in the development of Higher Still has much to commend it. The 5-14 programme in particular could benefit from a similar approach.

13. For the production of materials mentioned in 12. above, the Task Force might commission groups of 5 or 6 teachers who would work under the direction of the National Co-ordinator or a Development Officer.

Concluding remarks

With the start of a new era in the political life of Scotland, the SMC has a unique role to play with regard to mathematical education in Scotland. It is hoped that the items mentioned above will be of interest and will foster constructive debate. Other topics will be considered on another occasion.
If you wish to discuss the contents of this paper further, please do not hesitate to contact me.

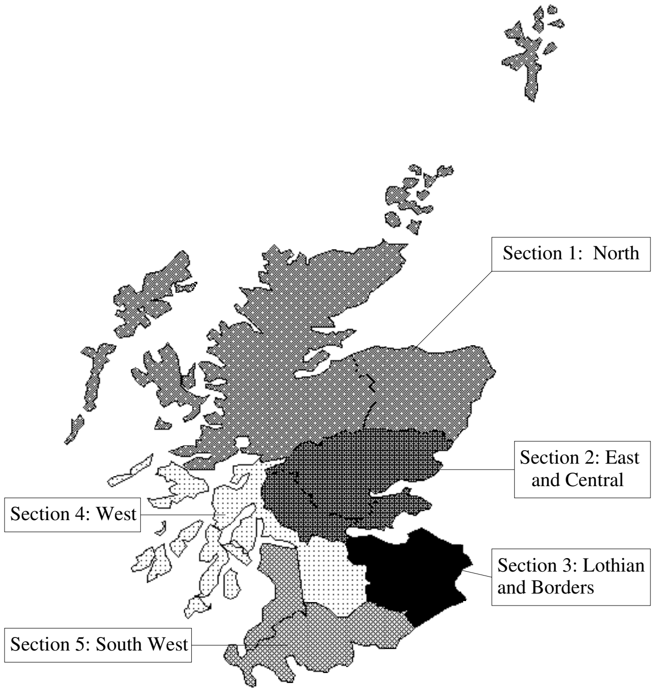
Professor A. C. McBride, OBE
Chairman
The Scottish Mathematical Council

Mathematical Challenge 1998-99

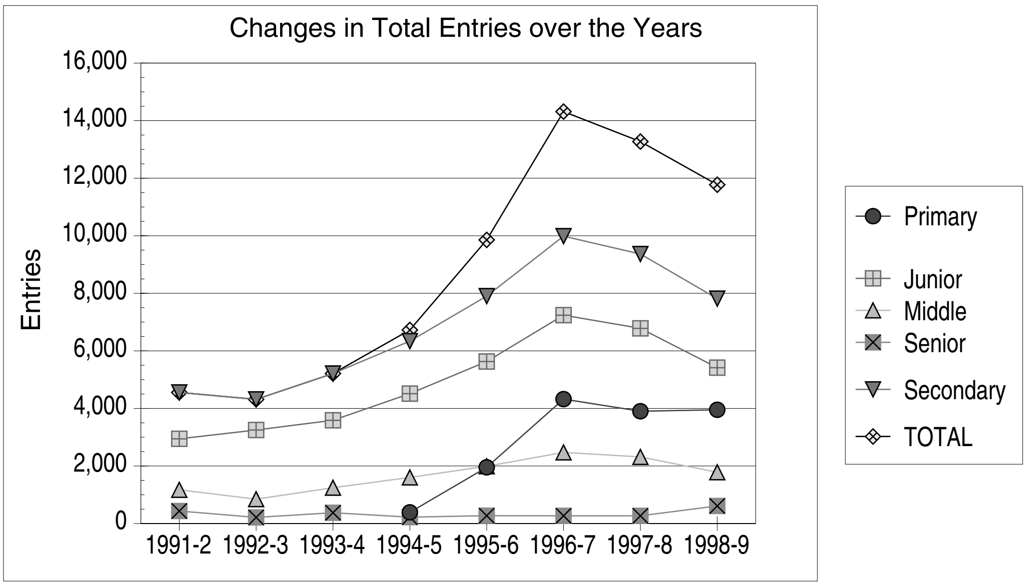
Session 1998-99 saw an administrative rearrangement of the Mathematical Challenge competition with five main sections instead of four. Sections 1 and 2 remain largely as they were before, but Section 3 has become slightly smaller by losing Dumfries and Galloway to the new Section 5. Section 5 has also taken over the southern part of the old Section 4. Section 4, still by far the largest section (in terms of number of entries), is split into a primary sub-section and a secondary sub-section. A report from each of the sections follows this general report.

The sections have adopted briefer, more manageable names and their approximate geographical locations can be seen from the following map of Scotland.

A further rearrangement of the competition is taking place in 1999-2000 with the secondary competition moving to two sets of five problems instead of three sets of four. This is partly to ease the administrative burden on both the Challenge organisers and on the teachers in the schools who kindly distribute and collect in the questions. It also recognises that secondary school pupils have many other calls on their time in the second half of the school year, usually resulting in a big drop in the number of entries to Problem Set III. The primary competition will continue to have three sets of three problems.



The numbers of entries appear to be up in the Senior Division in most sections, but Senior (S5 and S6) numbers are still low. The number of Primary entries continues at the high level of the last two years, although the overall number of Secondary entries has dropped back more towards its level of the mid 1990s. The quality of entries this year was generally high. An exceptionally pleasing solution to one of the Senior problems by a pupil of Boroughmuir High School, Edinburgh, is included after the section reports.



In reading the North Section report, readers may be dismayed to read Professor Patterson, for many years now the National Chairman of the Challenge, talking about his 'last report as organiser for the North Section'. Fear not – although he is retiring as North Section Organiser, he is continuing as National Chairman for a while at least. During the 1998-99 session, the National Committee welcomed various new members Hannah Fulford of Invergordon Academy has joined the National Committee. Frank Smith of St. Andrews University took over as Organiser for the East & Central Section and Elizabeth West of Paisley University has organised the new South West Section. Gerry McKaig of St.Andrews College, already a much valued National Committee member, became Organiser of the West Primary Section. At the end of the session, the Committee bade farewell and expressed their gratitude to David Weir, who had kindly stepped in to keep Section 4 afloat, and to Judy Goldfinch, the Deputy Chairman, whose term of office on the SMC had come to an end.

Early in the session, the Scottish International Education Trust and The Edinburgh Mathematical Society both agreed to renew their support for Mathematical Challenge for three-year periods. A successful application was made to the London Mathematical Society for a grant for 1998-99 and a kind offer from the Society of Petroleum Engineers proved to be very helpful. Professor L E Frankel of the University of Bath made another much appreciated donation. Other local sponsors are mentioned in the section reports. The appreciative comments made by Professor Patterson in his report below for the North Section, about the invaluable contribution of the markers, the question setters, and also of the teachers in the schools, applies to all sections – we really do appreciate your help: in fact, the competition could not continue without such support. Thank you all.

J M Goldfinch

Section 1: North

This is my last report as organiser for the North Section, which until very recently has always been known as Section 1. I am very pleased to say that Mathematical Challenge North Section continued to flourish in session 1998-99 and that it is now in the capable hands of Dr. Colin Maclachlan, Reader in Mathematics in the University of Aberdeen.

For the third consecutive year, the number of entrants in the section was high. The total was 958 compared with 985 in 1997-98 and 1010 in 1996-97. The following table of the numbers of entries and entrants in 1998-99 reveals some of the interesting details. The figures in italics give the totals for 1997-98.

Problems	Junior		Middle		Senior		Secondary Total		Primary		Overall Total	
I	352	*432*	124	*151*	88	*15*	564	*617*	331	*289*	895	*906*
II	222	*264*	86	*114*	58	*15*	366	*393*	239	*221*	605	*614*
III	177	*164*	69	*80*	46	*15*	292	*259*	211	167	503	*426*
Number of contestants	384	*489*	134	*173*	88	*15*	606	*677*	352	*308*	958	*985*

Number of schools taking part:	Secondary	62	*63*
	Primary	62	*58*

The reduction in the number of entries received in the Junior Division seems to be big enough to merit being called 'significant'. There was also a reduction in the Middle Division, which at first sight looks relatively smaller. However, the *percentage* reductions with respect to the numbers in 1997-98 (to one place of decimals) in the two divisions were

Junior Division	$((432 - 352) \times 100 \div 432 = 18.5\%$
Middle Division	$((151 - 124) \times 100 \div 151 = 17.9\%$

which suggests that the reduction in the Middle Division is relatively the same as in the Junior Division. And what about the figures for the Senior Division? In 1998-99 there were 88 entrants in the Senior Division of this section. Compare this with the figures for the seven previous years (working backwards from 1997-98) which were 15 20 21 18 28 18 26. Thus there were more entrants in the Senior Division in 1998-99 than there were in the four sessions from 1994-95 to 1997-98 inclusive. There were no evident reductions in standard, so that there were not only considerably more entries in the Senior Division, but also considerably more good entries. With encouragement from parents and teachers (already much in evidence) and perhaps some more tuition in how to tackle and to solve problems, standards could go yet higher. My message to S5 and S6 is this: congratulations on your efforts, keep responding positively to the challenge of the problems, always study the solutions which are sent out to schools in due course, so that you can learn as much as possible from the experience of taking part and, above all, never give in.

It has always been the case in Mathematical Challenge that a few entrants submit excellent work for Problems I, then disappear without trace. Why does this happen? Were the problems too easy? Did the problem sheets for later sets disappear? Reading the mark-sheets, it is distressing to see that there were some people who gained full marks in Problems I and then did not even submit one solution for Problems II.

Numbers of entrants who submit solutions for all the rounds in a competition like Mathematical Challenge can be illuminating. The following table gives (i) the total number of entrants E in a division or divisions, (ii) the number T who submitted entries for all three sets of problems and (iii) T as a percentage of E (i.e. 100T/E). The figures in italics are the corresponding figures for 1997-98.

Division	E, number of entrants		T, number of entrants submitting solutions to all three sets		T as a % of E	
Junior	384	*(489)*	148	*(135)*	38.5	*(29.9)*
Middle	134	*(173)*	66	*(71)*	49.3	*(41.0)*
Senior	88	*(15)*	45	*(15)*	51.1	*(100.0)*
All secondary	606	*(677)*	259	*(221)*	42.7	*(32.6)*
Primary	352	*(308)*	186	*(154)*	52.8	*(50.0)*
All divisions	958	*(985)*	445	*(375)*	46.5	*(38.1)*

Number of schools:

Secondary	62	*(63)*	49	*(48)*	79.0	*(76.2)*
Primary	62	*(58)*	44	*(33)*	71.0	*(56.9)*

Probably most of the entrants who submit entries to all the sets of problems hope to achieve recognition in the form of a certificate – gold, silver or bronze. From the above table, there were 384 entrants in the Junior Division of Section I in 1998-99, of whom 148 submitted solutions to all three sets of problems. As the table below shows, 96 of the 148 qualified for an award: 22 gold, 32 silver and 42 bronze. Expressed as percentages, 38.5% of entrants submitted solutions for all three sets of problems, with 25% of all entrants qualifying for an award; 5.7% of the entrants being awarded gold, 8.3% silver and 10.9% bronze. It must be stressed that in assessing the awards we do not base the numbers of prize winners on such percentages.

Certificates awarded in the North Section, 1998-99

Division	Gold	Silver	Bronze	Total
Junior	22	32	42	96
Middle	17	15	15	47
Senior	10	12	12	34
Primary	19	30	36	77
Totals	68	89	105	254

Of the 96 awards in the Junior Division, 40 were won by pupils in S1: 8 gold, 14 silver and 18 bronze. Of the 85 awards in the Primary Division, 9 were won by pupils in P6: 3 gold, 3 silver and 3 bronze. Two girls from Fraserburgh Academy attempted the problems in the Senior Division, even though one was in S3 and the other in S4. Both performed well; indeed the solutions presented by the girl in S3 were amongst the very best in the whole history of the competition. Mathematical Challenge does not aim to highlight young people of exceptional talent; nevertheless we are always delighted to see their work and we like to put on record the fact that the talent is there. Another significant award winner, from Robert Gordon's College, had, in his final year at the school, the satisfaction of qualifying for his sixth Mathematical Challenge mug; he was in the 'top class' for awards in each of his six years. Others have achieved this, but very few!

The award ceremony was held in the University of Aberdeen on Monday 14 June 1999. As in earlier years, Mathematical displays were on view in the Department of Mathematical Sciences, and again this proved to be very popular. The attendance exceeded our expectations, and those present included award winners from several places sufficiently far from Aberdeen to make the prospect of travel daunting, to say the least. Bayble School on the Isle of Lewis, the Sir E Scott School at Tarbert on the Isle of Harris, Kirkwall Grammar School, Plockton High School and Wick High School were some of the more distant places represented. Students' rooms in one of the Halls of Residence were used by some of those who had no option but to stay in Aberdeen for one or even two nights.

The Department of Mathematical Sciences was host for the day and provided lunch for 150. When heads were counted, there seemed to be more than this; we think that one or two hungry students, who possibly had been fasting over the examination period, may have wandered into the dining room by mistake. Everyone was welcome to attend Kenneth Brown's general talk on the attractions of Mathematics and the personal experiences of one who had become a Professor of Mathematics. He told us an interesting story of how a school teacher had misjudged the prospects of one of his pupils, which he assessed by estimating the length of the pupil's hair. We judged from the enthusiastic reception to the talk that the assessment had been inaccurate.

Professor Brown is a Vice-President of the London Mathematical Society, for several years one of the main sponsors of Mathematical Challenge. The North Section has its own supporters to help in local matters: a donation from BP Amoco helped to meet the expenses for travel and accommodation for people coming to Aberdeen to the award ceremony and having to cover the cost of long and expensive journeys. IBM UK Ltd., as in earlier years, financed the tea and biscuits that refreshed the crowd of about 200 between the talk and the presentation ceremony. Royal Mail continued to play an important part throughout the year in forwarding correspondence, including problems, solutions and mark-sheets, efficiently and courteously as well as post-free. We are greatly indebted to Royal Mail for this support, which, in financial terms, is equivalent to a handsome donation.

We owe a great deal to those who are prepared to spend long hours marking the entries. They have always been considerate in their awareness of the need to treat every entry seriously and evenly. When marking, credit must be given where it is due. A solution must not be treated as rubbish just because the method is much longer and more cumbersome than is necessary; if it is correct and works, then credit must be given. Thanks are also extended to teachers for their willingness to process Mathematical Challenge papers conscientiously. Much of what the National Committee for Mathematical Challenge does would not be possible without the assistance of teachers. Supportive parents are of course very important to us, but we do worry about the loss of talent that can arise when a promising pupil has to cope with a negative attitude, or even a hostile one, from home or from their peer group.

There are many others without whom Mathematical Challenge would not be able to operate. Those who suggest new problems, who have the interesting but time-consuming task of trying to do them and also to grade their suitability for the competition, are mostly very busy people, whose services are much in demand. It has to be remembered that a problem cannot be used unless we are sure about the solution. Ambiguities and errors in logic are not always obvious. We must ensure that there are no mistakes in our problem sheets or our solution sheets, since these are to be so widely distributed. The work is interesting but it can be stressful.

We thank all for their welcome support and encouragement. I hope that Mathematical Challenge, as a competition open to pupils in schools throughout Scotland and locally here in the North Section, will thrive for many years to come.

E M Patterson

Section 2: East and Central

The number of entries for the secondary school competition was broadly in line with last year. The expected growth in the Primary Competition has finally occurred with twice the number of schools and entries. The figures for 1998-99 (with those for 1997-98 in italics) were

Problems	Junior		Middle		Senior		Secondary Total		Primary		Overall Total	
I	641	*769*	221	*361*	79	*31*	941	*1161*	311	*142*	1252	*1302*
II	382	432	137	*138*	56	*20*	575	*590*	251	*130*	826	*720*
III	315	*291*	107	96	55	*16*	477	*403*	207	*163*	684	*566*
Number of contestants	705	*826*	227	*367*	80	*31*	1012	*1224*	345	*219*	1357	*1443*

Number of schools taking part:			
	Secondary	58	*60*
	Primary	58	*29*

In the secondary competition 407 pupils from 45 schools entered all three rounds; the corresponding figures for the Primary competition are 179 pupils from 39 schools.

Certificates awarded:

Division	Gold	Silver	Bronze	Total
Junior	17	22	37	76
Middle	26	33	27	86
Senior	8	8	17	33
Primary	22	30	50	102
Totals	73	93	131	297

The prize-giving was held at an Open Day in the University of St Andrews on Tuesday 1st June. 114 pupils from 28 schools were invited and there was a good attendance of pupils, teachers , parents and members of the local committee. The guest lecture entitled ‘A Mathematician’s Holiday’ was delivered by Dr John O’Connor of the University of St. Andrews. This highly illustrated presentation took us to the Alhambra in Spain to name but one tourist venue.

After lunch in New Hall, the prizes were presented by Pofessor Edmund Robertson of the University of St. Andrews. To finish the day, Dr O’Connor and other committee members led a Microlab session where one could design one’s own personalised wallpaper!

I would like to thank Dr Brian Fugard on behalf of the committee for all his hard work over the past two years as chairman and wish him all the best with his future career in Kent.

F I P Smith

Section 3: Lothian and Borders

This year saw a significant change to this section with the Dumfries and Galloway schools being incorporated into the new Section V. As a consequence, the number of secondary and primary schools taking part was roughly three-quarters of the previous year’s. This has meant the section has become a far more manageable proposition. One or two large schools failed to participate this year, which meant that the remaining numbers fell somewhat. Whether this was due to teething problems with the change or due to the number of markers these schools would be asked to provide is still unclear.

The numbers participating in the section this year were:

Problems	Junior	Middle	Senior	Secondary Total	Primary	Overall Total
I	353	124	23	500	243	743
II	255	77	20	352	242	594
III	218	80	13	311	239	550
Number of contestants	409	145	33	587	350	937
Completing all three sets:	154	50	10	214	148	362

Number of schools taking part:	Secondary	45
	Primary	32

A figure not often given is the percentage of schools that actually participate in the competition. For this region it is about 65% of the possible secondary schools and it would be nice to see this increase.

Our prize-giving was on Saturday June 19th in the Swann Building at the University of Edinburgh. Some 80 Gold and Silver award winning pupils together with 70 or so parents and teachers attended the ceremony with Dr Colin Aitken, Head of the Department of Mathematics and Statistics, presenting the awards. After refreshments a lecture “How to draw a Straight Line without a ruler” was given by Prof. Elmer Rees to the Middle and Senior pupils, together with parents and teachers. The Junior award winners participated in one of the two activities, “Bridges and Bouncing Balls”, or “Piles of Tiles”. The Primary sector were mailed their awards.

In each of the divisions the Gold cut-off was set at 87%, the Silver 73% and the Bronze 67%. For whatever reasons there were many high scores this year and the distribution of awards reflected this. Boroughmuir High School within Edinburgh distinguished itself by having the top award winners in each secondary division, quite an achievement.

The numbers of certificates awarded this year were:

Division	Gold	Silver	Bronze	Total
Junior	29	55	10	94
Middle	13	27	6	46
Senior	3	1	3	7
Primary	42	34	32	108
Totals	87	117	51	255

I am indebted to my markers and colleagues who have aided me in this enterprise: Thank you.

H W Braden

Section 4: West

Since this was the first year of the new Section 4, there is no way of comparing this year's statistics with last year. The final figures for this section are shown in the table below. Bracketed figures, for interest, are those of 1994-95, the last year for which I was previously responsible.

Problems	Junior	Middle	Senior	Secondary Total
I	763 (839)	221 (304)	74 (45)	1058 (1188)
II	481 (470)	140 (115)	41 (24)	763 (609)
III	401 (334)	105 (102)	31 (20)	537 (456)
Number of contestants	888 (947)	235 (315)	74 (45)	1197 (1307)
Completing all three sets:	308 (249)	101 (81)	31 (18)	440 (348)

Number of schools taking part: Secondary 70 (107)

You will note that, apart from first round Juniors and Middles (and of course numbers of schools), the figures for those local authority areas left in Section IV (Glasgow, Argyll and Bute, North and South Lanarkshire, and East and West Dunbartonshire) are higher than for the whole of the then Strathclyde Region of just four years ago.

A total of 266 certificates were awarded: 62 gold, 68 silver and 136 bronze. Gold certificates were presented at a ceremony held in the University of Strathclyde on Tuesday 8th June. The successful entrants were welcomed by Professor Adam McBride, who then presented their prizes and entertained them with a discourse on some interesting properties of numbers. After a buffet lunch, Dr David Weir gave a talk on the four colour problem.

In conclusion I must thank the large number (over thirty) of teachers prepared to help with the marking. Without them, running this section would be an impossible task.

D G Weir

Section 4P: West Primary

This new division consists of all primary schools in Glasgow, Argyll and Bute, North and South Lanarkshire, and East and West Dunbartonshire. The Division has its own organiser, administration and prizegiving.

Communication was therefore made directly with each of the 650 primary schools via the usual authority distribution channels. Primary teachers were invited to help as markers. However, only two teachers volunteered. They were invited to form a committee with the organiser to run the competition. Both accepted!

Marking was taken over most expeditiously by enlisting the services of 72 BEd students who had elected to specialise in mathematics education. This was overseen and monitored by their tutors.

A total of 840 pupils from 85 schools submitted entries to the sets of problems.

Problems	I	II	III	Number of contestants
Primary	600	578	423	840

19% of the entry received an award, distributed as shown:

Division	Gold	Silver	Bronze	Total
Primary	22	54	85	161

All gold and silver award winners were invited to a prizegiving ceremony in mid-June at St.Andrew's Campus, Faculty of Education, University of Glasgow. A talk on 'Codes' was given by Gerry McKaig and, after a short workshop (with refreshments), certificates and mugs were presented to all those pupils attending by Professor B J McGettrick, Dean of the Faculty of Education.

G McKaig

Section 5: South West

Although both the Primary and Secondary competitions in this new section (covering Ayrshire, Renfrewshire, Inverclyde and Dumfries & Galloway) were organised by the Department of Mathematics and Statistics at the University of Paisley, most of the Primary entries were marked by staff and students in the Faculty of Education at the Craigie Campus. This eased the burden of finding markers and of distributing entries for marking, and we are grateful to the staff and students who participated. Hopefully, the students learned something from the experience!

The numbers participating in the section this year were:

Problems	Junior	Middle	Senior	Secondary Total	Primary	Overall Total
I	424	151	13	588	341	929
II	239	77	8	324	267	591
III	197	68	7	272	235	507
Number of contestants				615	400	1015

Number of schools taking part: Secondary 50
Primary 55

The numbers of certificates awarded this year were:

Division	Gold	Silver	Bronze	Total
Junior	13	15	31	59
Middle	8	7	7	22
Senior	1	0	4	4
Primary	14	13	31	59
Totals	36	35	73	144

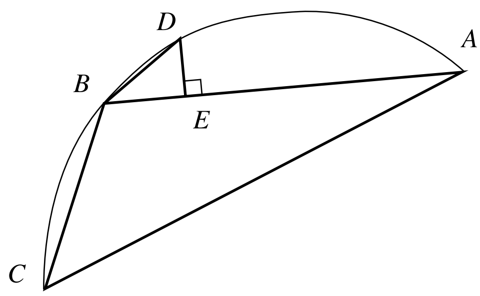
Since Section 5 covers such a large area, it was decided to hold award days at the University of Paisley's sites at Dumfries, Ayr and Paisley. Schools whose pupils gained awards were invited to bring along all interested pupils and their parents, and a total of over 200 attended the three events. Prizes were presented by the Principal of the University at Paisley, an Assistant Principal at Dumfries, and by the Dean of the Faculty of Education at Ayr. After the presentations, the Section Organiser, Elizabeth West, gave a talk entitled 'A First Encounter with Maps, Snowflakes and Crinkly Curves'. Soft drinks and a chance to chat rounded off a very pleasant afternoon on each occasion.

E West

An example of a novel and pleasing solution to problem S4 in Set III, 1998-99

The problem:

In the figure D is the midpoint of the arc AC of a circle, B is a point on the arc between C and D, and E is the foot of the perpendicular from D to AB. Show that E is the midpoint of the path from A to C along the line segments AB and BC.



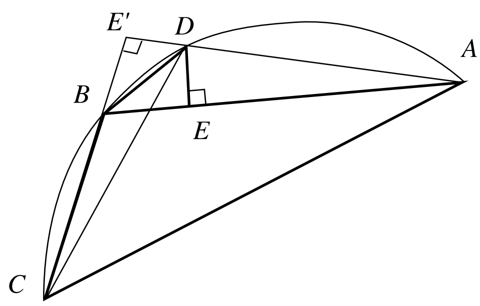
Solution from István Gyöngy (Boroughmuir H.S.):

Since D is the midpoint of the arc, $DC = DA$.

If we rotate the triangle DEA around D so that A goes to C, then line EA goes to line CB because the angles $\angle BCD$ and $\angle BAD$, standing on the same arc, are equal. The right-angled triangles BED and $BE'D$ are identical because they have the same hypoteneuse BD, and $E'D = ED$.

Therefore BE=BE'. Thus

$CB + BE = CB + BE' = CE' = EA$.



Mathematics Masterclasses

Adam McBride, University of Strathclyde
Based on a presentation at the SMC Conference, University of Stirling, 24 April 1999

Introduction

This article is in two parts. The first deals with the evolution and philosophy of masterclasses in mathematics. The second describes an actual masterclass for S3 pupils, as presented by the author over the last three years.

The development of masterclasses in mathematics

The concept of a masterclass is well established in the performing arts, notably in music; a group of talented students gather to study and perform under the direction of a "master". Masterclasses in mathematics may well have existed in the time of the ancient Greeks but the modern version seems to have evolved only fairly recently. The catalyst has been the Royal Institution of Great Britain based in London. The Royal Institution has organised Christmas lectures since 1826, the first series being given by Michael Faraday. For many years the subject matter was exclusively from the areas of physics, chemistry and biology. Mathematics first appeared in 1979 when the lecturer was Sir Christopher Zeeman. By then the lectures were being televised and many will have seen Professor Ian Stewart performing (with the help of at least one wild animal) in 1997.

Following the success of the Zeeman lectures, the then director of the Royal Institution, Sir George Porter, decided to launch a series of mathematics masterclasses. The first took place in London in 1981, again featuring Sir Christopher Zeeman. In the succeeding years there has been rapid development and now series take place in over 30 areas throughout the United Kingdom. In each case the target audience is made up of 12-14 year-olds with an interest in and enthusiasm for mathematics.

In Scotland there are series based on Edinburgh, Glasgow and the North of Scotland. The Edinburgh series has run for many years. Typically a programme of 8 classes is arranged during the autumn term. Schools around Edinburgh are invited to nominate pupils and 80-90 youngsters roll up on 8 successive Saturday mornings for 2½ hours of mathematical stimulation. Each meeting usually starts with a short exposition by the lecturer of the day which sets out the basic material upon which the rest of the presentation depends. After 30-40 minutes there is the first "workshop" involving activities based on the opening exposition. The lecturer is assisted by a number of teachers who act as tutors. (The teachers have received relevant material in advance.) At half-time there is a break for refreshments. Then comes further exposition and another workshop. At 12.30 it is time to go home but not without hand-outs and suggestions for follow-up and further reading.

Series have also been held in Glasgow (although rather more intermittently than in Edinburgh) with a similar format. That brings us to the North of Scotland where there have been some remarkable goings-on. From two reasonably natural bases in Aberdeen and Inverness, the show has gone out on the road to Skye, Lewis, Orkney and Shetland. In the last three years this programme has been augmented by a residential course for S3 pupils from all over the Highlands. Pupils arrive for lunch on Thursday and leave around 4 p.m. on Friday. In a little over 24 hours, they get 3 masterclasses, 2 lectures, puzzles and competitions served up by a team of 6 "lecturers" and a similar number of tutors. This all comes about thanks to the enthusiasm and indefatigableness of Emeritus Professor Edward Patterson (University of Aberdeen) and Mr George Gibson (Adviser, Highland Council).

As one who has been heavily involved in the delivery of masterclasses, perhaps I may be allowed to make a few comments. (*These are my own views and do not necessarily represent the views of any institution, society or other body with which I am or have been involved!*) I am seriously concerned at what has been happening in S1-S2 to pupils showing an aptitude for mathematics. Pupils of that age are usually keen to learn. Abler pupils (not just the high fliers) relish a challenge. They need to be s-t-r-e-t-c-h-e-d as far as possible so that they see the fascination and beauty of mathematics. Otherwise their interest in the subject is snuffed out. I often wonder how many youngsters are lost to mathematics at this stage in their development. This is no criticism of the hard-pressed classroom teachers who do an excellent job within the system. However even the best teacher only has 24 hours in each day so that extension material from other sources is needed to provide support. Masterclasses offer such material. For an indication of the wide variety of possible topics, the reader should consult [2].

One of the beauties of mathematics is that some of the hardest problems are easy to explain, even to youngsters. Fermat's Last Theorem is a case in point. With a minimum of jargon, it is possible to get down to serious business quickly. My own experience shows that you can set ambitious targets and succeed. Is it reasonable to try to explain the continued fraction for $\sqrt{2}$ to 13-year-olds? YES, it is.

With a few prompts and leading questions, they can cope. (See below for more details of this!) This is only one example but plenty others could be cited. In summary, we should aim high. The abler pupils can be s-t-r-e-t-c-h-e-d a long way. They will survive and be much the better for the experience.

A masterclass on Diophantine Equations

I shall now describe a masterclass given to S3 pupils from all over the Highlands. The pupils were put into groups of around 20. I repeated the class three times during the residential course with a different group each time. The response varied from group to group but in all cases the pupils did well. The time allocated was 1¾ hours. The material split naturally into two parts, with a break of 10 minutes between them for a breath of fresh air.

Part 1

Diophantine equations are algebraic equations in which the variables are integers. They are named after Diophantus of Alexandria who studied them in detail around 250 AD. To start the session I display equations such as

$$x - y = 1; \quad x + y = 1; \quad 2x + 2y = 1;$$
$$x^2 - 2y^2 = 1; \quad x^2 + y^2 = z^2; \quad x^3 + y^3 = z^3.$$

I ask the pupils to tell me how many solutions there are (none, one, finitely many or infinitely many) when the variables are all **positive** integers. We then consider what happens when we allow zero or negative integers. (Regarding the last item on the list, the pupils have had a lecture which mentions Fermat's Last Theorem.) After at most 10 minutes, the pupils start on the following problems.

1. I went into a post office and bought some 19p stamps and some 25p stamps. The total cost was £ 2.88. How many stamps of each type did I buy?
2. You have an unlimited supply of 19p and 25p stamps. Determine the largest total cost of postage which **cannot** be made up from these stamps.
3. You have an unlimited supply of 19g and 25g weights and a balance. Explain how you can weigh out any whole number of grams exactly.
4. You have two containers which have a capacity of 19 litres and 25 litres respectively. How can the containers be used to measure out exactly 1 litre of liquid?

5, Repeat each of the previous problems with the numbers 19 and 25 replaced by 20 and 26 respectively.
Comment on what happens in each case.

Commentary on part 1

Q1 is quickly dealt with. For Q2 I supply a worksheet with the integers from 1 to 513 arranged in rows of 19. Numbers which can be produced from 19 and 25 are scored off systematically. First we ignore 25 and delete all multiples of 19 (easy because of the layout of the integers on the worksheet). Then we use 25 once, 25 twice and so on. The pupils soon get into the swing of things and end up with a largest number not scored out. I ask them how this number is related to 19 and 25. On several occasions over the years I have received the correct answer, namely $19 \times 25 - (19 + 25)$. Q3 usually proves easy (with a prompt required occasionally). We do Q4 together on the OHP. The pupils shout out the "moves" and again usually spot an algorithm. Regarding Q5 I give them a hint (useful for all exams); the examiner doesn't ask the same thing twice!

Of course, the problems may look daft from the practical point of view. There aren't too many 19 gram weights or 19 litre jugs kicking around! Nevertheless, the purpose is served of studying $19x + 25y$ and $20x + 26y$ in four different contexts. For a detailed investigation into the postage stamp problem, consult [1].

Part 2

Suitably refreshed after their short break, the pupils launch themselves into a study of $x^2 - 2y^2 = 1$, one of the six equations in the initial list. I present this to them in the form of a structured investigation, as follows.

We shall try to find **non-negative integers** x and y such that

$$x^2 - 2y^2 = 1.$$

(i) Can you spot any pairs x and y which work? How many such pairs do you think there are?

(ii) Extend the following table by spotting patterns

x	1	1	3	7	17	
y	0	1	2	5	12	

For each pair (x, y) in your table work out $x^2 - 2y^2$. Review your answer to (i).

(iii) For each pair (other than the first) in your table in (ii), work out x / y to 6 decimal places. What do you think is happening?
(It may help to square each of your values for x / y.)

Commentary on Part 2

In (i) one or two pairs are usually spotted. Most pupils feel there are more. This is confirmed in (ii) which is usually well done, albeit with the help of calculators. For (iii) pupils are advised to write their answers in alternate columns so that the trend is easier to spot. The suggestion of squaring is sometimes needed but not always; some pupils know $\sqrt{2}$ as a decimal. To finish off the session we work together on the OHP. After checking that their algebra is up to scratch, we convince ourselves that

$$\sqrt{2} - 1 = \frac{1}{\sqrt{2} + 1} \qquad \text{(exactly!).}$$

Then the fun starts. We obtain successively

$$\sqrt{2} = 1 + (\sqrt{2} - 1) = 1 + \frac{1}{\sqrt{2} + 1} = 1 + \frac{1}{2 + (\sqrt{2} - 1)}$$

$$= 1 + \cfrac{1}{2 + \cfrac{1}{\sqrt{2} + 1}} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + (\sqrt{2} - 1)}} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\sqrt{2} + 1}}}$$

$$= 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + (\sqrt{2} - 1)}}} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\sqrt{2} + 1}}}} \quad \text{etc.}$$

We may write

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}}$$

which gives us the simple (in a technical sense!) continued fraction for $\sqrt{2}$. If we truncate the decimal expansion for $\sqrt{2}$ at each decimal place we get successively $1, \frac{14}{10}, \frac{141}{100}, \frac{1414}{1000}, \ldots$ This is a sequence of **rational** numbers converging to the **irrational** number $\sqrt{2}$. If we similarly truncate the continued fraction obtained above, out come the rational numbers

$$1, \quad 1 + \frac{1}{2}, \quad 1 + \cfrac{1}{2 + \cfrac{1}{2}}, \quad 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}} \quad \text{etc.}$$

Work these out and you'll see where the table in (ii) comes from! The youngsters seem to lap this up. For homework they are invited to investigate $x^2 - 3y^2 = 1$ and $x^2 - 4y^2 = 1$ (bearing in mind that we don't ask the same thing twice!). With that, an action-packed 1¾ hours comes to an end. It's time for another break before I do it all over again with the next group. The pupils have been s-t-r-e-t-c-h-e-d, I'm exhausted but everyone is happy! The effort is well worthwhile.

References

1. A. Gardiner, *Discovering Mathematics: The Art of Investigation*, O.U.P., 1987, 0-19-853282-2.
2. M. Sewell (ed.), *Mathematics Masterclasses : Stretching the Imagination*, O.U.P., 1997, 0-19-851493-X.

The 1999 International Mathematical Olympiad

Adam McBride, University of Strathclyde

Introduction

The 40th International Mathematical Olympiad (IMO) was held in Romania during the period 10-22 July, 1999. The fortieth anniversary of the competition provided an appropriate reason to return to the place where the first IMO was held in 1959. (The alert reader should immediately ask why this was not the 41st IMO. The explanation is that there was no IMO in 1980 because of unforeseen difficulties.) In the intervening 40 years the IMO has changed considerably. For example, the original 7 countries have swollen to 81, thereby producing a truly international event. The one part of the world still not well represented seems to be the continent of Africa, which can only muster teams from Morocco, Tunisia and South Africa. Another change concerns the difficulty of the problems being set. The very first problem in the first IMO asked students to prove that

$$\frac{21n + 4}{14n + 3} \text{ is irreducible, for all positive integers } n$$

where “irreducible” means “in its lowest terms”. If you notice that

$$3(14n + 3) - 2(21n + 4) = 1$$

the proof is a one-liner (Why?). Contrast this with the six problems listed below.

This year the U.K. team had a new Leader, the appropriately named Dr. Imre Leader (University College, London). I went with him as an Observer to try to explain the inner workings of the Jury and other items with which he was unfamiliar.

Selecting the U.K. IMO team

In a previous article (SMC Journal 27, 1997, pages 11-14), I described the various competitions (and their associated acronyms) which form the basis of the selection process. As usual, things kicked off in November 1998 with the U.K. Senior Mathematical Challenge (UKSMC) which attracted well over 40000 entries. The paper seems to have been found fairly easy with a good number of high scores. As a result, schools entered around 1200 pupils (as opposed to the usual 800) for BMO 1, the first round of the British Mathematical Olympiad, held in January. It was unfortunate that BMO 1 proved much harder than normal and many low scores were recorded. Around 100 pupils went forward by invitation to BMO 2 in February and this paper, while again quite tough, produced a much more acceptable range of scores. Exactly 20 pupils were then selected for a 4-day residential training session at Trinity College, Cambridge in April. The session concluded with the Final Selection Test, a 4½-hour paper with just 3 questions, each of IMO standard. A squad of 8 students then embarked on a correspondence course during which the final team of 6 emerged.

Team:	Thomas Barnet-Lamb	(Westminster School)
	Rebecca Palmer	(Clitheroe Royal Grammar School)
	Marcus Roper	(Northgate High School, East Anglia)
	Oliver Thomas	(Winchester College)
	Oliver Wicker	(Cockermouth School, Cumbria)
	Jeremy Young	(Nottingham High School)
Reserves:	Stephen Brooks	(Abingdon School)
	Michael Spencer	(Lawnswood High School, Leeds)
Leader:	Imre Leader	(University College, London)
Deputy Leader:	Richard Atkins	(Oundle School, nr. Peterborough)
Observer:	Adam McBride	(University of Strathclyde)

Notice that all eight students come from schools in England. Where are the bright sparks from Scotland, Wales and Northern Ireland? What about the girls? Even although we had a girl in the team, there were very few in the last 100 who sat BMO 2. I have highlighted these difficulties in the past. See, for instance, my report on the 1998 IMO (SMC Journal 28, 1998, pages 13-16). We must continue to try to rectify the situation.

It’s time to let you see some problems. Have a go at them yourself. More importantly, give them to your bright pupils in S4-S6 and see what happens! I present first a couple of BMO problems before showing the papers set in the IMO.

BMO 1 1999, Q2
A circle has diameter AB and X is a fixed point of AB lying between A and B. A point P, distinct from A and B, lies on the circumference of the circle. Prove that, for all possible positions of P,

$$\frac{\tan \angle APX}{\tan \angle PAX} \text{ remains constant.}$$

BMO 2 1999, Q4
Consider all numbers of the form $3n^2 + n + 1$, where n is a positive integer.
(i) How small can the sum of the digits (in base 10) of such a number be?
(ii) Can such a number have the sum of its digits (in base 10) equal to 1999?

The Problems of the 1999 IMO

All contestants sat two papers on consecutive days.
Each paper contained three problems, each problem being worth 7 points.
On each day the time allowed was 4½ hours.

FIRST DAY

1. Determine all finite sets S of at least three points in the plane which satisfy the following condition:
   for any two distinct points A and B in S, the perpendicular bisector of the line segment AB is an axis of symmetry for S.

2. Let n be a fixed integer, with $n \geqslant 2$.
   (a) Determine the least constant C such that the inequality
   $$\sum_{1 \leqslant i < j \leqslant n} x_i x_j \left(x_i^2 + x_j^2\right) \leqslant C \left(\sum_{1 \leqslant i \leqslant n} x_i\right)^4$$
   holds for all real numbers $x_1, \ldots, x_n \geqslant 0$.
   (b) For this constant C, determine when equality holds.

3. Consider an $n \times n$ square board, where n is a fixed **even** positive integer. The board is divided into n^2 unit squares. We say that two squares are *adjacent* if they have a common side.

   N unit squares on the board are marked in such a way that every square (marked or unmarked) on the board is adjacent to at least one marked square.

   Determine the smallest possible value of N (in terms of n).

SECOND DAY

4. Determine all pairs (n, p) of positive integers such that
   p is a prime,
   $n \leqslant 2p$,
   and $(p - 1)^n + 1$ is divisible by n^{p-1}.

5. Two circles Γ_1 and Γ_2 are contained inside the circle Γ, and are tangent to Γ at the distinct points M and N, respectively.

   Γ_1 passes through the centre of Γ_2.

   The line passing through the points of intersection of Γ_1 and Γ_2 meets Γ at A and B.

   The lines MA and MB meet Γ_1 at C and D, respectively.

   Prove that CD is tangent to Γ_2.

6. Determine all functions $f : \mathbf{R} \to \mathbf{R}$ such that
   $$f\left(x - f(y)\right) = f\left(f(y)\right) + xf(y) + f(x) - 1$$
   for all $x, y \in \mathbf{R}$.
   (Here, $\mathbf{R}$ is the set of all real numbers.)

You are invited to send in solutions, enclosing an SAE please, to
Adam McBride, Department of Mathematics
University of Strathclyde, Livingstone Tower
26 Richmond Street, GLASGOW G1 1XH.

Some Statistics

The papers proved to be hard and scores were low.

The top individual score was 39 (out of 42).

The top team score was 182 (out of 252).

The U.K. team came 20th with a score of 100.

Individual scores were as follows:

Thomas Barnet-Lamb	19	Silver Medal
Rebecca Palmer	16	Bronze Medal
Marcus Roper	14	Bronze Medal
Oliver Thomas	21	Silver Medal
Oliver Wicker	10	
Jeremy Young	20	Silver Medal

Although the IMO is, strictly speaking, an individual competition and, officially, there is no team competition, considerable interest still attaches to team totals. For the record, here are the top 20 teams with their totals out of 252:

182	China, Russia	177	Vietnam	173	Romania
170	Bulgaria	167	Belarus	164	Korea
159	Iran	153	Taiwan	150	USA
147	Hungary	136	Ukraine	135	Japan
130	Yugoslavia	116	Australia	109	Turkey
108	Germany	107	India	104	Poland
100	United Kingdom				

Conclusion

There is much more to an IMO than the two 4½-hour papers. Aside from all the hard work, there is the opportunity for young people from all over the world to make new friends and soak up the culture of another country. An excursion to the castle of Vlad the Impaler (aka Dracula) went down well, as did alfresco picnics and various forms of musical entertainment. A mixture of good weather, good company and pleasant surroundings added up to a memorable experience for us all.

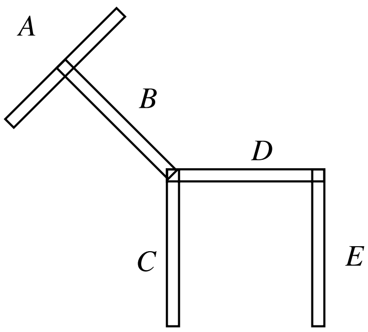
The UKMT and its competitions
Some of my favourite questions from the Challenges

Bill Richardson – Elgin Academy.
Based on presentations made at the SMC conference and the Joint MA/ATM conference in Liverpool, both during April 1999.

By now, the UKMT and its numerous Challenges (Junior, Intermediate, Senior and their sequels) should require no introduction. In the presentations referred to above, I indulged myself by scanning the first ten years of the Challenges and selecting questions which took my fancy. The selection is idiosyncratic and says much about what I find appealing! During the first ten years the Challenges were developing and growing rapidly. From 1988 to 1993 there was a Schools Mathematical Challenge with an age range of roughly 11 to 15. From 1994 there have been the Junior Mathematical Challenge and the Intermediate Mathematical Challenge which, in Scottish terms, are for up to and including S2 and up to and including S4, respectively. The UKMT also runs the Senior Mathematical Challenge but that came from a different background and formed no part of the talks.

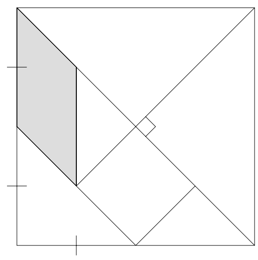
Altogether I picked some 32 questions from the 425 available. It was a very difficult choice! Clearly, I can include only a few in this Journal but I hope you find them as entertaining as I do.

1990 Schools Challenge 3
13.
The diagram shows a reindeer made from five matches. You have to move just one match to make another similar reindeer. Which match should you move?



A B C D E

1991 Schools Challenge 4
19.
The seven pieces in this 12cm × 12cm square make a Tangram set. What is the area of the shaded parallelogram?



A 6cm² B 12cm² C 18cm² D 36cm² E 144cm²

[This looks as though it needs a lot of calculation but there is a nice dissection.]

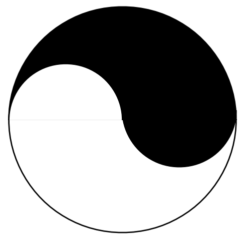
1992 Schools Challenge 5
14.
Gill's back! This year was her fourth birthday. The highlight of her party was a game of musical chairs. The game got down to herself, nurse, and me. Only two chairs were left – the hard chair and the comfy chair, with a big gap between them. The music stopped and we all piled onto the nearest chair, some on top of one another. If Gill's bottom was firmly in contact with one of the two chairs, in how many different ways could this have happened?

A 4 B 6 C 8 D 10 E 12

1994 Junior Mathematical Challenge 1

12.

The diagram is made up of one circle and two semicircles. Which of the three regions – the black region, the white region and the large circle – has the longest perimeter?



A the black region B the circle

C the white region D black and white are equal and longest

E all three perimeters are equal

1995 Junior Mathematical Challenge 2

8.

What is the units digit of the product of 12345679 and 63?

A 2 B 3 C 7 D 9 E can't be sure without a calculator

1998 Junior Mathematical Challenge 5

15.

At the first ever World Worm-Charming Championship, held at Wollaston, Cheshire in July 1980, Tom Shufflebottom charmed a record 510 worms out of his 3m × 3m patch of ground in 30 minutes. If the worms, of average length 20cm, stopped wriggling and were laid out end to end round the edge of his patch, approximately how many times round would they stretch?

A 8½ B 9 C 20 D 30 E 510

1994 UK Intermediate Challenge 1

18.

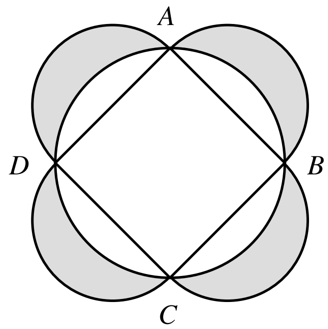
Which is the largest number?

A 1^{500} B. 2^{250} C. 3^{100} D 4^{75} E 5^{50}

1997 UK Intermediate Challenge 4

23.

The square *ABCD* is inscribed in a circle of radius 1. Semicircles are drawn with diameters *AB*, *BC*, *CD*, *DA* as shown, and the parts of these semicircles which lie outside the original circle are shaded. What is the total area of these four shaded regions?



A $3\pi + 2$ B 2 C π

D 1 E $\frac{\pi}{2}$

So that's it. I have not included my favourite, you will have to ask me when you see me!

For more information about the UKMT you can look at the web site

www.mathcomp.leeds.ac.uk

telephone 0113 233 2339 or write – UKMT, Department of Mathematics, University of Leeds LS2 9JT.

In 1999, the UKMT produced a Year Book. It was sent to all participating schools but extra copies can be purchased from the Leeds office (£5 including postage).

Reaching the parts . . .

Those who read Journal 28 might recall an item which I wrote about the World Federation of National Mathematics Competitions (WFNMC) Congress in China in 1998. Along with many other activities, I gave a copy of Mathematical Challenges III to the editor of the WFNMC Journal. The review below appeared in Volume 12 number 1 1999.

Mathematical Challenges III
Scottish Mathematical Council,
66 pp., paperbound, A4 size, 1998
ISBN 0-9532786-0-3
£UK 6.50

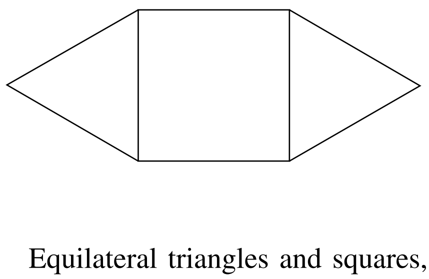
Mathematical Challenges III is the third in a series of books containing problems and solutions from the Scottish Mathematical Council's problem-solving competition Mathematical Challenge. The Challenge itself has grown from 2841 secondary student entries in 1993-94 to 7426 in 1996-97. The Challenge was also extended to primary students in 1996-97.

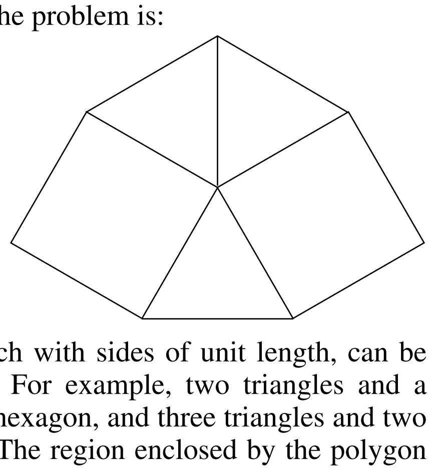
Mathematical Challenges III contains all ninety questions from the 1994-95, 1995-96 and 1996-97 Mathematical Challenge competition. Within each year there are problems for Junior, Middle and Senior students, with Primary problems added in the 1996-97 section. On reading the questions it is obvious why the competition itself has grown. Students who take part are exposed to a wide variety of interesting and unusual problems which are both accessible and challenging.

The introduction to the book discusses the aims of the competition and the importance of problem-solving. It lists the aims as being to encourage young people to be enthusiastic about mathematics, to appreciate its importance, to understand how it works and to develop their powers of reasoning in an enjoyable and satisfying way. Any young person attempting the problems in this book would certainly be challenged to develop their powers of reasoning, and could not fail to find the experience enjoyable and satisfying.

The problems themselves are imaginative, varied and engaging. They cater for a wide range of ages, and cover the full range of mathematical topics normally encountered in school mathematics courses. They invite both exploration and rigour, allowing almost all students to attack the problems in a meaningful way, while still challenging the very best problem solvers.

I particularly like Question 28 from 1995-96, partly because it is from the rapidly declining but vitally important area of geometry, but also because it embodies the very best principles of a mathematical problem. The problem is:





Equilateral triangles and squares, each with sides of unit length, can be used to construct convex polygons. For example, two triangles and a square can be put together to form a hexagon, and three triangles and two squares to form a 7 sided polygon. (The region enclosed by the polygon must be covered exactly by the triangles and squares used in the construction.) What is the maximum number of sides of a convex polygon which can be constructed in this way? Describe how the process can be used to construct a convex polygon with 11 sides.

I will not spoil the reader's enjoyment by providing the solution, but what I like about the problem is that:

- It is accessible in a very hands-on exploratory way.
- It is a somewhat surprising result that there is, in fact, a maximum number of sides.
- The Aha! moment that characterises so much of problem-solving is very much in evidence.
- The proof of the final result is delightfully elegant.

Mathematical Challenges III contains ninety such problems that both entertain the reader and provide an extremely valuable resource for the classroom. Permission has been granted for the problems to be copied for individual or classroom use, provided the source is acknowledged. Of course, the inches and pounds would need to be changed for those using the problems in more enlightened countries. Perhaps the only suggestion I would make to the authors is that there be some cross-referencing to topic areas such as number, geometry, rates, measurement, counting and so on. However, many of the best problems defy classification, and any teacher worth their salt will quickly decide which problems are appropriate to use at any given time.

I thoroughly recommend the book as a great source of enjoyable and challenging problems for teachers and students of all levels from upper primary to late secondary.

Steve Thornton,
Director Australian Mathematics Teacher Enrichment Project,
Australian Mathematics Trust,
University of Canberra ACT 2601,
E-mail: Steve T@amt.canberra-edu.au

Isn't it nice to be appreciated!!

Bill Richardson, Elgin Academy

Prizes for CSYS mathematics

In 1999, as in the two previous years, prizes have been awarded to candidates for ‘outstanding performance’ in the CSYS mathematics papers. The prizes are given on the basis of the performance in an individual paper which means that there is a prize for each of the five papers with there being more prizes for the General Paper than the other papers. There were more prizes awarded in 1998 than in 1997 with the total amount of money distributed being £760 which was also the pattern in 1999. Financial support in 1999 was provided by The Scottish Life Assurance Company, Thomas Nelson (Publishers), The Nationwide Building Society, The Edinburgh Mathematical Society and also a private donation. As well as a cheque, each prizewinner receives a certificate and a (mathematics) book generously donated by Oxford University Press.

Winners in 1999 were:–

CSYS Paper 1
John Mackay (Firrhill High School); Iain Drummond (Denny High School);
Euan Kerr (Auckinleck Academy); Yung Man (Hyndland Secondary School);
Stewart McGrenary (Stonelaw High School); Gavin Dunn (Gryffe High School);
Alison Coton (Greenock Academy).

CSYS Paper 2
Robert William Hunter (Westhill Academy); John Mackay (Firrhill High School); Alexander Douglas (George Watson's College).

CSYS Paper 3
Christopher Waudby (Bearsden Academy); Neil Brown (George Watson's College).

CSYS Paper 4
John Mackay (Firrhill High School).

CSYS Paper 5
Iain Drummond (Denny High School); Alan Smith (Hermitage Academy).

Glasgow's Numeracy Improvement Programme

A Report of the Presentation given by Patricia Brown and Fiona MacDonald at the Scottish Mathematical Council Conference, Stirling University on Saturday 24th April 1999.

We started our presentation by explaining the background to Glasgow's Numeracy Improvement Programme which is part of the Early Intervention Initiative. The original project consisted of:

1 Educational Psychologist whose role was to assess the success of the project
2 Community Education Workers whose remit was family literacy and numeracy
5 Nursery Tutors whose remit was originally literacy but latterly became numeracy
12 Primary Literacy Tutors
2 Primary Numeracy Tutors

All were practising professionals on a 23 month secondment.

The Numeracy Improvement Programme was developed in response to the S.O.E.I.D. document, ‘Improving Mathematics Education 5 - 14’, taking into account the findings of T.I.M.S.S., A.A.P. and recommendations arising from recent H.M.I. reports.

The programme was presented using a ‘Coaching in Context’ approach, where tutors use demonstration lessons to model suggested practice. It started in January 1998 and is continuing until June 2000.

At the start of the programme it was agreed that the main focus for the programme would be mental mathematics. We therefore looked at the 5 - 14 Mathematics Guidelines concentrating on Strands and targets which said ‘mental’ or ‘mentally’ in them. Taking these as our guide, and using a wide variety of mathematical texts, we devised a framework for the teaching of numeracy from Level A to Level E (Level F is now being added).

We explained to delegates that the framework covered three main areas:

Targets
Interactive Teaching
Resources

We broke down the targets of 5 - 14 into smaller steps, giving teachers examples of teaching and resources that could be used to achieve the target. To illustrate this to delegates we used an overhead for Level B, part of which is shown below

Target	Interactive Teaching	Resources
Add and Subtract		
0 - 20		
addition	doubles / near doubles	coat hangers / pegs

One of the key elements of our programme was that the resources we used should be inexpensive and readily available. We demonstrated to delegates how simple wire coat hangers and pegs can be used to reinforce linking facts and doubles.

To reinforce the concept of interactive teaching, we taught the delegates a game at this point. This was a very straightforward game using a nine cell grid which could easily be differentiated using different dice. We allowed delegates some time to play the game which generated discussion and turned the lecture room into a normal sounding primary classroom. Some delegates were surprised to discover the variety of dice readily available in catalogues.

This activity led naturally on to a description of calculating strategies, teaching strategies and activities we demonstrate in the classroom. At this point we were asked to clarify the calculating strategies we would be teaching. These are listed below;

- doubles and halves
- + / - 10, multiples of 10
- + / - 9, 11 by +10 and compensating by 1
- making 10
- bridging through 10, multiples of 10
- use of place value building on responses - scaffolding learning

The teaching strategies we use to achieve the above were;

- use of open-ended questions
- modelling correct mathematical vocabulary

- encouraging children to verbalise their thinking – provide an ethos where it is acceptable to be wrong
- build on responses - scaffolding learning
- collaborative working, pupil - pupil, teacher - pupil, parent - pupil
- high expectations of all pupils

At this point we asked delegates to participate again by completing a multi-cultural 100 square jigsaw. Again this led to much discussion and even some bewilderment as delegates realised that it was not straightforward. The logic behind this activity was to make delegates look at and discuss the patterns, not merely to try doing it by fitting the shapes together as children often do. It also reinforced the fact that the use of games and puzzles provides children with a motivating environment where they use and extend skills and knowledge previously taught. Those delegates who successfully completed their jigsaws we are sure would agree. Delegates were pleased to be given a handout containing copies of the four jigsaws and we hope that pupils throughout Scotland have enjoyed them.

Having experienced some of the activities, delegates realised the importance of discussion and participation for effective learning to take place. We emphasised at this point that although we worked at the lower stages of the primary school, it was important that all staff were aware of these approaches to numeracy. To reinforce this we had developed a numeracy support pack which had resources suitable for all stages in the primary. Two of the main components of this pack are a series of number bond flash cards and cards for washing lines. We illustrated this by using another overhead showing Level D.

Target	Interactive Teaching	Resources		
NMM		washing line		
fractions	recognise equivalence	0	0.5	1
	between fractions and	0	1/2	1
	their relationship to	0	50%	1
	decimals			
	order fractions, decimals			
	and percentages	0.01		1 square

Delegates were also interested to discover the different ways in which 5 - 14 targets are met in a primary context.

During the short question time at the end of our presentation we were asked about implications post primary. In response we acknowledged that we had no first hand experience of teaching post primary seven but that In-service delivered to secondary colleagues in Glasgow had been well received.

Other questions related to the availability of Glasgow's framework, support pack and other resources.

In conclusion we stressed again that throughout the Numeracy Improvement Programme we have encouraged teachers to reflect on their own practice. We have also tried to generate an interest in numeracy and show children and teachers that it can be fun. Our enthusiastic approach to the teaching of numeracy has succeeded in making this a well received programme. The assessment of the programme has shown that teachers have appreciated someone working alongside them in their own classroom but have also felt confident about continuing these strategies for themselves.

We Aim to Praise

Mark Doherty, St. Andrew's High School and Ian Barnett, Inverkeithing High School

This article is based on the address given at the SMC Conference held at the University of Stirling, on the 24th April 1999.

It has been well recognised that a positive ethos within a classroom is desirable and can, if effectively used, lead to increased attainment, through the positive expectations of the teacher and the pupils. Creating such an environment is therefore seen to be a key aspect of a teachers day to day classroom management.

Creating an Ethos of Achievement by Rewarding Positive Behaviour.

Positive expectations are the key to success.

Everyone can win.

There are many ways of creating this positive ethos and perhaps the most effective is through the assumption that "everybody likes to receive praise." The aims and objectives of creating this positive environment through the use of praise can be summarised as follows

(1) To increase pupils' expectation of themselves and others.

(2) Recognise the achievement of the individual.

(3) To create a positive environment, which can be used as a basis for repairing difficult situations.

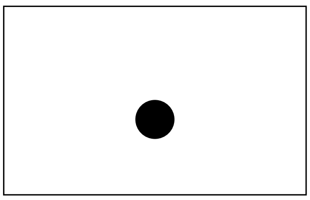
(4) To make the pupils more positive about maths; not "maths is fun" but "being in maths is fun", (i.e. rewarding).

(5) To encourage pupils to take more responsibility for their own learning and actions.

(6) To enhance/enable successful teaching and learning.

The overall aim is therefore to improve the attainment of every pupil in the class.

We must always 'think positive'. The diagram below represents a common misconception about behaviour in schools. People claim that they see a black dot in the rectangle. This is true, but the dot only takes up a relatively small area of the rectangle.



Only a minority of pupils will actually misbehave. Yet we as teachers often spend more time dealing with the indiscipline instead of recognising the good discipline/attainment of the majority of the class. Thus by concentrating on good attainment and behaviour we are making more effective use of our time with pupils in the classroom. We as maths teachers know that if we half the radius of a circle we are effectively quartering the area. A small effort can make a big difference.

The quality of the class matters little. What matters is the person who is taking it.

Outlined below are two strategies which were developed within a maths department of a school which had a "challenging clientele".

STRATEGY 1 – The Raffle

A raffle was run amongst the S1, S2 and S3 classes. If pupils did well with a piece of work and/or behaved particularly well they would get a raffle ticket. The teacher would identify a condition for getting a raffle ticket at the start of each lesson. Pupils were unaware of the conditions previous to the lesson.

e.g. "Pupils who have brought their own pencils get a raffle ticket."
OR
"Pupils who do their work and don't disturb others."
OR
"Listen carefully to the teacher during the lesson."
etc.

The aim of this strategy was to reduce the incidence of low level indiscipline. It was decided that a raffle ticket could not be taken from a pupil if she/he fulfilled the set condition. Thus, often, classes had two or more conditions to contend with. Pupils put the raffle tickets, with their name on the back, into a box at the end of the class. Any exceptional behaviour or work was rewarded with a raffle ticket regardless of the conditions and any pupil who quite obviously tried really hard and persisted to achieve success was rewarded. The more raffle tickets you collected the better chance you had of winning a prize. At the end of approximately four weeks, a raffle was drawn for each class and prizes given out.

The results from this strategy were quite encouraging. For example, previous to applying this strategy, around seven pupils failed to bring in pencils to class. By the end of the four weeks only the occasional pupil forgot a pencil. Perhaps one of the most startling results of this strategy came after the four week trial and into a period where the usual praise strategies such as verbal praise and written praise in jotters was being used. It became apparent that pupils had learnt, through the use of simple conditions set at the start of each class, what the teacher expected of them and by fulfilling these conditions the pupils expected praise from the teacher. Thus the positive expectations of both parties served to create and maintain a positive ethos within the classroom. The pupils had learnt the expectations in a fun way rather than the traditional "you will do...... or else face the consequences..." way.

STRATEGY 2 – 10 Stars Cause For Praise

In addition to verbal praise, written praise in a jotter and happy faces which are always welcomed by pupils, the use of a materialistic form of praise almost has a value all of its own to the pupil. This strategy involved pupils collecting stars, which would be put by the pupil, or teacher, on to a star chart on the wall of the classroom beside the pupils' name.

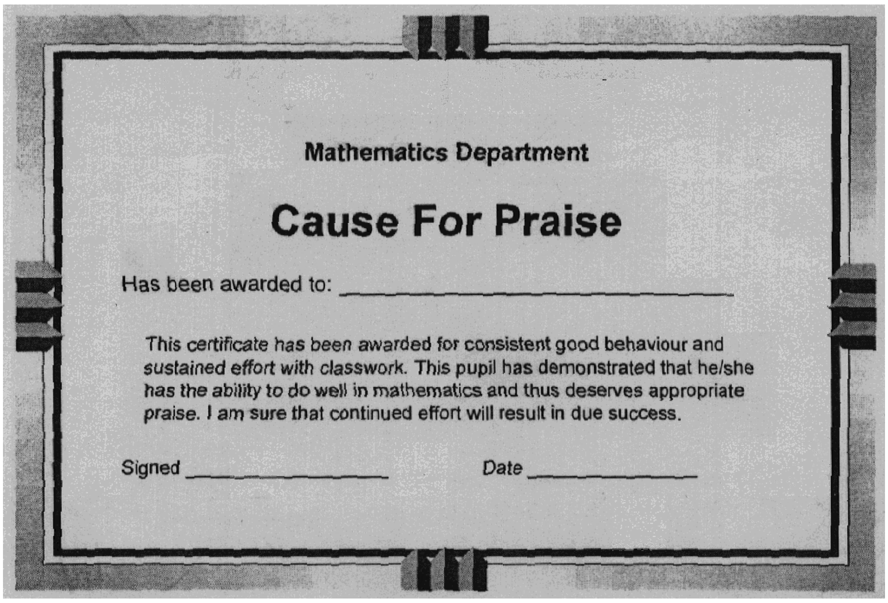
The condition for getting a star depended on consistent good behaviour and good work of a pupil. Each teacher that used this strategy awarded a star at his or her discretion. There was no fixed condition to adhere to. Instead, the strategy concentrated on the individual. Each individual had to show their class teacher that he/she deserved a star. This system aimed to reduce the incidence of misbehaviour by giving pupils a chance to prove themselves and if they did, they got a reward. Thus instead of an incident resulting in a detention or referral it may well end up with an opportunity for praise and thus a reward.

Once the 10 stars were obtained by the pupil they received a certificate as shown below. The time limit for achieving the 10 stars was around 12-15 periods. Stars were given out each period that the teacher saw the pupils.

This strategy proved to be very effective. What made it effective was that it concentrated on the individual. Again pupils learnt what was expected of them and often concentrated on their strengths, and tried to overcome their weaknesses in order to gain a star. It also meant that those pupils who consistently did well anyway, were pushed that little bit further in order to gain the star. This strategy, more so than the previous strategy, was aimed almost exclusively at raising attainment.

The use of effective praise strategies had a twofold effect on the classes. Firstly is improved the pupils behaviour (the incidence of interval detention, the main sanction available to the teacher, reduced by 70%). Secondly it improved attainment.

The fact that the results of these strategies were seen as being so significant by the Board of Studies that it decided to implement similar strategies throughout the school is a credit to the members of the mathematics department involved. One small step can lead to giant leaps forward. Good practice should always make policy.


Mathematics Department

Cause For Praise

Has been awarded to: ______________________________

This certificate has been awarded for consistent good behaviour and sustained effort with classwork. This pupil has demonstrated that he/she has the ability to do well in mathematics and thus deserves appropriate praise. I am sure that continued effort will result in due success.

Signed ________________ Date ______________

At St. Andrew's High School, where I am currently teaching, a praise strategy already existed before I arrived. It is based on a stamp/merit system where pupils receive stamps for effort/attainment. Three stamps lead to a merit in the homework diary and three merits lead to a departmental "Good News" letter being sent home. There are also opportunities to gain departmental award certificates and whole school certificates. Unlike the strategies mentioned above, which only applied to the Mathematics Department, this strategy is whole school policy and is applied in every department. Thus St. Andrew's High not only aim to create a positive ethos within the classroom but try to create it throughout the whole school.

The use of praise strategies to teach positive behaviour in a fun way can be summarised by Aristotle

We are what we repeatedly do.
Excellence then, is not an act but a habit.

The importance of praise in the school environment can be highlighted by referring to HMI Performance Indicators.

H.M.I. Performance Indicators

> ".......Praise is used effectively to encourage pupils and to build their confidence."

> ".......The importance of praise as a motivating and positive aspect of school life is well understood and the use of praise permeates all aspects of the life of the school. Pupils have high expectations of themselves and others."

This part of the talk shared ideas about teaching methods, based on Lee Canter's "Positive Discipline", which have been effective in several comprehensive schools in England and Scotland. The emphasis is based on the philosophy that people thrive on praise rather than criticism.

Specific examples of difficult pupils were shared indicating the success of the techniques not only from a whole class point of view but also focusing on the individual as well.

Fife Council Psychological Services state:-
"Students have the right to a teacher who will set firm and consistent limits." and hence I make a point of going over classroom rules (which are displayed on the wall) with any new class. These are rules that I feel comfortable with and have been adapted depending on the school. It is important to highlight that

it is the choice of the pupil to misbehave and hence their choice to accept the consequences. The rules tell them what is required of them in the class. They are not threatening and allow pupils to respect the rights of others.

MY CLASSROOM RULES
* FOLLOW INSTRUCTIONS WHICH THE TEACHER GIVES YOU.
* REMAIN SILENT AND LISTEN WHEN THE TEACHER OR PUPIL IS TALKING TO THE CLASS.
* NO SWEARING, TEASING OR SHOUTING OUT.
* KEEP HANDS, FEET & OBJECTS TO YOURSELF.
* NO EATING , DRINKING OR CHEWING.

Various praise strategies are used in the classroom including verbal praise, stamps, certificates, positive referrals (and, when teaching in England, phone calls homes). These are displayed on the classroom walls alongside the Consequences. This is a hierarchy of six warnings showing what will happen to disruptive pupils if they choose to misbehave in class.

PRAISE RECOGNITION	CONSEQUENCES (for present school)
* VERBAL PRAISE.	* WARNING.
* STAMPS AND CERTIFICATES.	* TIME OUT.
* INFORM P. T.	* WHITE PUNISHMENT EXERCISE.
* INFORM GUIDANCE.	* YELLOW PUNISHMENT EXERCISE.
* INFORM B.O.S.	* REFERRAL.
	* WITHDRAWAL.
	SEVERE DISRUPTION WITHDRAWAL.

Positive Discipline allows me to have my say in class without getting anxious and rules can be restated to remind pupils if they are doing something wrong. There is still a place for anger as highlighted by Aristotle

"Anyone can get angry, that's easy. But to get angry with the right person, to the right degree, at the right time, for the right reason, in the right way, is not so easy."

The use of stamps and certificates is well recognised and the following quote from Brian Boyd and Paquita McMichael highlights this:–

"The certificates ..... help motivate and recognise achievement. They are appreciated by parents."

Positive expectations are the key to successful classroom behaviour management. Most pupils can behave. Those pupils who do not behave either choose not to, or haven't been taught.

When a teacher believes that pupils can choose to behave, his or her expectations are raised, pupil attainment is raised.

SCCC Graphic Calculator pack

Maureen McKenna, Kilsyth Academy

The Scottish Consultative Council on the Curriculum (SCCC) are committed to supporting the Higher Still initiative and are working with the Higher Still Development Unit (HSDU) to try to provide support material for all subjects presenting Higher Still qualifications.

The ACME (Advanced Calculators in Mathematical Education) recommended, amongst many other things, that there was a need for material to support the teaching of mathematics with technology. In response to this, SCCC agreed to produce teachers' notes and student exercises to support the use of the graphic calculator in the classroom within the Higher Still courses.

The material, which has been produced, includes topics from the Intermediate 1, 2 and Higher courses. The final pack will be more than 150 pages. Hopefully, by the time this article is published the pack will be out in centres.

The pack consists of brief teachers' notes; Calculator Skills Sheet(s) linked to student exercises, Calculator Hint Sheets, 'Quick Tour' sheets and programs. The Calculator Hint Sheets cover all the basic functions of a graphic calculator, such as how to draw a graph, how to use the table facility, etc. It may be initially that a class set of these sheets will be needed, but as expertise grows a small number should only be needed for students to refer to. The 'Quick Tour' sheets are aimed more at teachers and describe the functions and the menus on the graphic calculator.

Up to this point, I have avoided referring directly to the make of calculator which is used within this pack. The decision to select a specific make and model of calculator was not an easy one and one which was not taken lightly. It would have been impractical to write material which referred to more than one model of calculator, the concern being that the mathematics would 'get lost'. We decided to choose the Texas TI-83. This calculator was felt to be affordable whilst also having a reasonable 'shelf-life' – that is, it will not be out of date within the next few years. We were also aware that Texas had already committed itself to developing the teaching of mathematics with technology by the T^3 initiative.

However, we are very conscious that the decision to choose the TI-83 will not please everyone, (I add here that my own school did not have any TI-83s!). Other manufacturers have been in discussion with SCCC and perhaps in future the same pack will be republished with the references to the TI-83 replaced by another make. In order to minimise the impact of writing the material for one make of calculator the references to the TI-83 have been kept to the Calculator Skills Sheets. This should allow those teachers who are able to, to rewrite the Calculator Skills Sheets for their make of calculator.

The next steps for the writing team is to consider the production of more material, perhaps for the statistical content of Higher Still courses and for topics at Advanced Higher, and of course, to monitor the use of the first pack.

What is the appropriate role of hand held Computer Algebra Systems (CAS) in the teaching and learning of secondary school Mathematics in Scotland?

Eric Brown, Ian Forbes and Tom MacIntyre
The University of Edinburgh

Eric Brown, Ian Forbes and Tom Macintyre are Lecturers in Mathematics at The Faculty of Education of The University of Edinburgh (formerly Moray House Institute of Education).

In the Autumn of 1996, Bert Waits and Frank Demana, Professors at Ohio State University and founders of the Teachers Teaching with Technology (T3) programme, commented that "the portability and affordability of this tool (the TI 92) will mean the teaching and learning of mathematics will change significantly in the next few years." Since then the price of a TI 92 has dropped from £200 as it was then, to nearer £100, and the TI 89 (with CAS but not Cabri) is now with us. They also report that "some traditional paper and pencil skills will continue to be necessary... However, we must stop spending large portions of our time teaching obsolete paper and pencil algebra and calculus manipulations. This is our challenge for the future."

In 1998 the Scottish Consultative Council on the Curriculum (SCCC) published a paper for discussion and consultation on 'Advanced Calculators and Mathematics Education.' One of the recommendations was for the Council of the SCCC to "encourage detailed research into the potential benefits and problems associated with using graphics calculators and, in particular, CAS calculators, to ensure that future curriculum development in this area is well informed."

The Faculty of Education of the University of Edinburgh decided to take up the challenge set by Waits and Demana and the SCCC, and in August of last year put forward a research proposal and application for central support. The application was successful and Texas Instruments agreed to support the project by providing the TI 92's. The University agreed to fund the time for the research and the Scottish Office Education and Industry Department (SOEID) agreed to provide funding to be used as an incentive for participating schools. Finally, the University of Edinburgh Development Trust also provided a Small Project Grant for the study.

CAS systems have been available on PC or Macintosh for many years. As a result their impact on mathematics has been largely limited to university level. Advances in technology have resulted in 'advanced calculators' like the TI 92, which have computer algebra software, built in, and are powerful, portable and user friendly. The aim of this project is "to gather information on the use of Computer Algebra Systems in the teaching and learning of mathematics in a situation where whole classes of students have regular access to them in hand-held form.

This study aims to identify areas of the current curriculum where the use of CAS is appropriate. In particular we hope to identify ways of developing 'symbol sense' and thus enable students to tackle algebraic work with greater confidence as well as addressing the issue of student confidence with the use of CAS technology and making staff less 'computer shy'. The study will focus on the Mathematics Units leading towards the Higher Course award from the Higher Still programme currently being implemented in Scottish schools and colleges. Achieving data on the use of technology balanced by pencil and paper methods is an expected outcome of the study. The aims of the research will be complementary to those of similar studies being carried out in Norway, Austria and the USA but will focus on the Scottish curriculum.

The research is taking place in six large comprehensive secondary schools in the City of Edinburgh, Midlothian, Fife and West Lothian Councils. Three of the schools will be given equipment and fully supported by the research team and three will be non-intervention 'control' schools. The criteria for selection were based on locality, size of the mathematics department and the number of pupils presented for Higher Grade Mathematics. The selection and matching of schools involved in the study was based on data obtained from the Scottish Qualifications Authority (SQA) in conjunction with SOEID Audit Unit statistics. An initial sample of schools was obtained using the Higher School Characteristic Index (HSCI) from the Audit Unit's target setting documentation - based on records of performance in Higher Grade examinations and the proportion of pupils entitled to free school meals. As this information was not specific to mathematics, it was necessary to access the school level of data to match schools on their records of performance in Higher-Grade mathematics over the last three years. This resulted in our final sample of three pairs of schools, matched by HSCI, general performance in upper school examinations and specific performance in mathematics over recent years. Two teachers delivering the Higher Course of study in Mathematics have also been selected from each school. It is expected that one of the chosen teachers will act as 'contact' person, as a lead teacher, promoting the use of CAS in a positive manner. The second teacher may well be less convinced or enthusiastic, but prepared to use the technology as fully as possible in the delivery of the course. An expected sample population of between 150 and 180 students will ultimately be involved in the project. Texas TI 92 models of CAS calculators will be provided and loaned to each student for their dedicated use throughout the duration of their Higher course - i.e. until June 2000.

The project has four phases over the period from Autumn 1998 until the Summer of 2000. Phase one was the identification of schools and teachers/groups of students to take part in the study and the obtaining of access through the local authority and senior management. Having done this, staff had to be familiarised with the technology and trained in appropriate use of TI 92. Finally in this phase, time had to be spent identifying and agreeing areas of the curriculum to target with CAS input and approaches to be taken.

The second phase is to 'pre-test' the target groups to ascertain their algebraic skills and confidence level at the start of the project, prior to being introduced to CAS. Students will be expected to work until they have completed all they can do or until the end of the allocated time. The 'test' will give a starting point and benchmark, covering algebraic skills expected of students at this level (and beyond). Alongside each question is a space for students to comment and to indicate how confident they feel with the work, allowing a measure of the student's attitude towards mathematics generally and algebra in particular, to be gauged. Once this has been done, arrangements will be made for the hardware to be put in place and to familiarise students with the operating system.

During the third phase, staff from the Faculty involved in the research will provide tutor support for the schools; team-teach, observe and evaluate lessons on agreed areas of the curriculum.

The fourth and final phase of the research will be to conduct a series of face-to-face interviews with staff and a sample of students. Towards the end of the project and before the Higher 'Course Assessment' (Higher Still provision), all students will sit an algebraic skills 'test' of a similar nature to the one sat at the beginning. Some of the same questions will be utilised, but in addition, algebraic work covered during the course of study will also be explored. This will also give an opportunity to measure student abilities and attitudes at the end of the period of using CAS and an opportunity to consider progress made in skills level and algebraic confidence by the pupils in the control schools compared with those directly supported through the use of CAS. All students will also be asked at this final stage to complete a questionnaire on the different aspects of the study. Once this has been completed there will be an analysis of the interviews and a final evaluation of the study. On completion of this final analysis, the findings will be reported, leading to publication of the study in Autumn of 2000.

There are a few potential difficulties with the research. Since CAS calculators are currently disallowed in examinations, and are likely to remain so for the duration of the project, they will be used in the teaching and learning only. On ethical grounds, it may be necessary for the researchers to provide TI 82 or TI 83 calculators for examination purposes. These alternative calculators are permitted in examinations and have similar operating systems but no CAS. There may also be difficulties in terms of release of staff and use of staff time. It is hoped that the project can be directly linked to school development plans, target setting and a general commitment to improving standards. The link with improving numeracy and algebraic standards, currently a priority in mathematics education, will help ensure that there is a strong school commitment to the project.

The context of the study and the perceived benefits which participants will gain, through greater access to the technology, will hopefully make these issues less of a problem. An indication of the commitment and confidence of the schools involved has been reflected in the move by one of the intervention schools which opted to extend the study by funding the purchase of calculators for a third class. We look forward to reporting on progress made with this study in the next SMC Journal.

Some Issues Arising from the Proposals on Advanced Calculators

Chris Pritchard
McLaren High School, Callander

In August 1998, following an extensive consultation exercise, SCCC published proposals on the use of graphics and advanced symbolic calculators in the middle and senior school years in the discussion paper *Advanced Calculators and Mathematics Education*. The proposals are here welcomed as a sound and realistic basis for the introduction of these powerful new aids to mathematics teaching, though there are a number of issues which clearly need further consideration.

Issue 1: Divisions among Mathematics Teachers

There are few subjects which divide the body of mathematics teachers so cleanly as the use of calculators (and ICT more generally) in the classroom. The two camps have taken up entrenched positions, agreeing only on the central role of mental methods. Broadly speaking, those in favour of calculators have held sway, and quite properly so, but the concerns of the more conservative minority group should not be taken lightly.

Issue 2: In-built Bias to Consultation Exercises

It is incumbent on the sponsors of change to seek wide approval for their proposals. Contrary to the express wishes of the report's authors, ACME was initially sent to local authorities, not to schools. Not only did this have the effect of restricting access - a state of affairs that led inevitably to the extension of the consultation period - but it likely shaped the initial responses, for when questionnaires are sent out to local authorities with proposals to implement imaginative curricular change they are likely to be completed by educationalists with a sympathetic philosophical outlook. Many of the advisers who typically owe their appointments (at least in part) to their curricular innovations are precisely those to whom local authority directorates delegate the task of responding. Similar comments may be made about the responses from teacher education establishments. Therefore, we must tread carefully, despite the positive response the Review Group were given, as indicated in §1.3.

In the light of worrying international comparisons (Kassel Study, TIMSS) SOEID has signalled a return to more circumspect practice with regard to the use of calculators in the early years [1]. Two of the four bodies it acknowledges as having influenced its revised position are the leading proponents of reduced calculator usage in secondary teaching (CIMT, Exeter University) and primary teaching (Barking and Dagenham Education Authority). Further afield, the DfEE [2] has issued the results of a large survey of English schools revealing that a staggering 42% of heads of mathematics departments believe that ICT contributed little or nothing to mathematics teaching [3]. These observations are made not in an attempt to argue the conservative line but to emphasise the need to carry both camps, those excited by the prospect of using CAS (computer algebra system) instruments in their teaching and those who believe they will further erode the skills base.

Issue 3: Resource Implications

Section 2 offers an excellent, concise argument in favour of the latest technology together with suitable cautionary advice. The international study (§ 2.2) reveals a number of different patterns of calculator usage and although there are comments about schemes for the purchase of calculators an apparent link between the ability to purchase and the need to purchase (because of usage in lessons and assessments) is surprisingly not drawn. It is noticeable that of the four countries in which graphics calculators are integrated compulsorily - Norway, Denmark, Sweden and Portugal - the first three have high per capita incomes. Norway may indeed be ready and willing to allow unrestricted use of symbolic calculators in the near future but in Scotland, where average per capita income is low in comparison, substantial public financing would be essential.

The matter of resourcing is taken up again in Section 4. Nowhere are the current prices of graphics calculators or symbolic calculators specified. Many schools already have a stock of graphics calculators but if, as *Higher Still* is implemented, their use is to be extended, albeit in a phased programme, from

CSYS and Higher to Advanced Higher, Intermediate II, Higher, and possibly Credit Levels (§ 4.1) the cost to local authorities would be very significant. With CAS calculators near the £100 mark the notion that parents would be able to buy them, even with discount schemes in place, is wholly unrealistic. There may well be a tiny minority of schools where many parents can contribute but there will be many more where none are in a position to do so. Any requirement that parents pay is likely to be socially divisive. The burden on local authorities is likely to be far higher than the proposals seem to suggest.

Issue 4: Support for Teachers

On the face of it, there can be little doubt that calculators of all sorts do, indeed, reduce the time spent on routine, repetitive tasks allowing more time to be spent reflecting on underlying principles. The success of recurrence relations as a Higher topic has depended on graphics calculators and, in the future, the opportunity within the Advanced Higher programme to use symbolic calculators to integrate functions by parts using iterative methods and to elicit coefficients of correlation and regression in statistics will excite many teachers. Those who have already explored the possibilities of teaching using the TI-92 will be aware of the complexity of the instructions and the level of expertise required to make use of the technology.

To be fair the proposals stress the need for a comprehensive programme of training (§ 4.3). Among mathematics teachers there will be many who will view with trepidation proposals to incorporate what amount to hand-held computers in mathematics. They may dread doing so within the domain of statistics, an aspect of mathematics they may be teaching for the first time. If this is so, it is worrying that the opportunity for teachers to tap into the expertise of the Statistics in Scottish Schools Project (led by John McColl at the University of Glasgow) has been restricted, in the main, to teachers from schools within striking distance of Glasgow and with senior managers with an enlightened view of staff development. As early as May 1996, the project was delivering in-service training on the use of the rather cumbersome TI-92 in computing regression and correlation coefficients as well as means and standard deviations. By the summer of 1999 it had been pressed into service the more user-friendly advanced calculator, the TI-83, to calculate confidence intervals.

If such expertise has been somewhat spurned under the *Higher Still* development programme it must be questioned whether and to what extent the familiarity with the new technology will develop within the profession. It is one thing to identify six major needs to be addressed just three pages before the end of a 22-page consultation document (§ 4.4) but another to bring such an adventurous project to full fruition, as frankly admitted in the final paragraph.

Concluding Comments

The arguments in favour of extending the range of technology to mathematics teaching are well-made in the proposals. There are considerable, though not insurmountable, hurdles ahead, not least in the short term, promoting the proposals to those who are hesitant for technophobic or genuinely held didactic reasons. There are many teachers who would wish to see full bibliographies published in proposals such as these. Hopefully, references to articles and books on calculators in mathematics education could be incorporated in any support materials which follow. It seems that teachers of mathematics, from nursery teachers helping infants to produce simple computer images of people constructed of squares, triangles and circles, to university lecturers whose students can evaluate triple integrals at the push of a button, we are all confronting essentially the same challenge. How can we employ the increasingly fantastic gadgetry to enhance the learning of the subject without consigning justification and proof to books on the history of mathematics education? This may be the greatest of the challenges we face as we move towards the third millennium.

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2 *DfEE Statistical Bulletin Issue No 3/97* (March 1997).
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Are Boys Performing as Well as they Ought?

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There was a time when boys did better in mathematics than girls. Indeed, mathematics was deemed a boy's subject. It was thought that differences in brain function and the type of play in which they were involved in early life contrived to make 'the average' boy necessarily more able in mathematics. The Chelsea Studies in the 1970s pinpointed areas of greatest discrepancy in performance - for example, on problems requiring good spatial perception - and influential educationalists, backed by perspicacious publishers, responded by developing new teaching materials. The gender problem became a focus for a number of groups and professional organizations who investigated motivational factors, teaching methods and the implicit bias of teaching materials and examination papers. In time, and quite likely as a result, the girls crept past the boys. And so now we are beginning to think about how to improve the attainment of boys, crucially without reducing the performance of girls. This is an exploration of the issues, together with some tentative thoughts on amelioration.

Prior to session 1998-99 at McLaren High School, mixed-ability teaching had been the norm for more than a decade. A setting arrangement was put in place for S2 pupils in August 1998. The new groups were made up strictly on the basis of performance in S1. There was a significant gender imbalance in the top set: girls 65%, boys 35%. There appeared to be issues worthy of further investigation, the first amongst them being whether the imbalance was peculiar to mathematics within the school.

It was not possible to make any comparisons among departments with respect to performance in S1 or S2. In fact the only data available were the Standard Grade results which, as part of the school's quality assurance exercise, had already been entered onto a spreadsheet by the end of August. It was not a difficult task to produce grade point averages for the boys and the girls. The table below gives the difference in grade point average between the sexes for each subject, a positive figure indicating the extent to which girls did better than boys on average. The figure in the second column is the number of candidates taking each subject, a figure which gives a measure of credence to any conclusion drawn. Consequently, subjects with fewer than 20 candidates have not been included.

Art & Design	0.8	42	German	– 0.1	37
Biology	– 0.2	53	History	0.4	73
Craft & Design	– 0.1	35	**Mathematics**	**0**	**117**
Chemistry	0.1	60	Music	0.1	27
Comp. Studies	0	38	OIS	1.5	29
English	0.2	117	PE	– 0.4	25
French	0.5	64	Physics	0	53
Geography	0	45	***Overall***	***0.3***	***117***

There are gender differences of note in favour of girls in Art & Design, French, History, OIS and, bearing in mind that the whole year group tackled the subject, possibly English too. Broadly speaking, there is parity between the sexes in Biology, Craft & Design, Chemistry, Computer Studies, Geography, German, Mathematics, Music and Physics, and an imbalance in favour of the boys (probably insignificant because of the small presentation numbers) in PE.

The apparent neutrality of the statistics for mathematics could have brought our enquiry to an end but it would have been a premature end. Instead, we sought out information on the gender imbalance across Scotland and discovered that the school figures paralleled the national trend. The whole-school difference of 0.3 (albeit given to a single decimal place) coincided with that for the country as a whole when the Scottish Examination Board analysed the figures for 1994 [1], the only year for which such an analysis has been performed. In fact, the conclusions drawn by the SEB were uncannily applicable to our data:

> At Standard Grade, girls tended to do better in their examination results overall than boys: 0.30 of a grade better in the all subject average for 1994. When the subjects are grouped together by mode, girls achieved 0.50 of a grade better than boys in the creative subjects, 0.45 of a grade better in the language and communication mode, 0.37 of a grade

> better in the social subjects, and 0.38 of a grade better in the technology mode. The mode averages in science subjects, mathematical subjects, and Religious Studies were all near zero. The only subject in which girls did consistently worse than boys was Physical Education.

It is also interesting to note that one major way in which girls outperformed boys, according to the national data for 1994, was by doing especially well in the internally assessed elements, notably *Investigating* in the social subjects, *Speaking* in modern languages, *Investigating* in Mathematics. (Similarly, it was the girls' poor showing in the *Practical Performance* which let them down in PE.) With the internally assessed elements since or soon to be abandoned in many subjects, including Mathematics, will this automatically depress girls' results?

The Scottish Qualifications Authority has already undertaken a large-scale data-crunching exercise and discovered that theoretically, except in a tiny number of special cases, the final grade awarded to a candidate on the basis of taking the mean grade for the two elements and rounding up where appropriate, should be precisely that otherwise gained from all three elements. Empirical evidence from England and Wales would appear to back up this position. Historically, south of the border, girls have performed especially well on internal assessments (project or investigation) which have formed up to 50% of the final award at GCSE. Yet, the recent reduction in the weight given to the internally assessed element has not depressed girls' results relative to boys.

We were unaware of performance data from further afield but came across interesting information in a booklet produced by the Secondary Heads Association [2]. The headteachers reported that:

> Underachievement is not peculiar to either sex but there are thought to be four times as many underachieving boys as underachieving girls. In England, there was a surge in the performance of girls (and hence a relative decline in the performance of boys) from the early 1990s onwards. In 1993 girls outperformed boys in the GCSE examinations for the first time and they have repeated this feat every year since. Even in their traditional strongholds of mathematics and science boys have seen their performance matched by girls.

A key feature of the problem appears to be the anti-intellectual, anti-educational attitude and behaviour which for far too many boys pervades their lives both inside and outside school. Females constitute one-seventh of those educated in units for pupils with behavioural difficulties and just one-tenth of the adult prison population. Parents unwittingly encourage the dichotomy between the sexes by promoting among their daughters the sedentary, educationally beneficial activities of reading and discussion whilst persuading their sons to engage in active sporting pursuits which are beneficial educationally in only a limited way. It is not surprising, therefore, that for girls language and communication skills are more readily acquired, though perhaps the more telling consideration for general educational development is the positive attitude to learning that is being fostered quietly among girls over a number of years even prior to their arrival in scondary school. Factors affecting attitude include:

Girls
- value clear presentation and expression of their work
- are better organised
- can concentrate better
- prefer direct teaching and adopt a more passive learning style
- underestimate their abilities and work harder to compensate

Boys
- prefer active learning to soaking up what the teacher utters
- overestimate their abilities whilst fearing failure and the derision of their peers
- often believe that regardless of their talents and efforts they will not secure a good job after school
- suffer from a paucity of male role models

In the light of these points it seems appropriate to reiterate the need for teachers, including mathematics teachers, to (i) vary their teaching style (ii) offer advice on organizational skills, both general and subject specific (iii) create a learning environment in which pupils feel confident that a wrong answer will not elicit rebuke or scorn. Principal Teachers might consider whether their allocation of teachers to classes is likely to intensify or attenuate the 'male role model' factor.

From a list of fifteen initiatives suggested in the headteachers' booklet a small number have the potential to counter the gender imbalance in performance, among them:

(i) challenging the "It's not cool to achieve" attitude of boys with gender issues explicitly discussed at parents' evenings, at open evenings under the ægis of the PTA or the School Board, and at assemblies for boys only

(ii) praising and rewarding older boys privately rather than at public ceremonies, so there is no loss of face with peers bent on mocking success

(iii) seeking greater rigour from teachers with respect to boys – teachers have lower expectations of boys, accept a lower level of presentation from them and more readily extend deadlines for them.

As a department we have begun experimenting with a fourth measure. A school in Essex has had some success in raising the attainment of boys by arranging classes in a such a way that every boy sits next to a girl, every girl next to a boy. This device has the effect of breaking up coteries of poorly-motivated boys. Our first impressions have been favourable.

The gender imbalance in performance in Mathematics is becoming a pressing issue but as yet (SCRE take note) it is somewhat under-researched. As a school and as a department we have made a start but there is more to be learnt and more to be done if boys are to perform as well as they ought.

References

1 Scottish Examination Board, *Gender and SCE Examinations (SEB Research 3)*, 1995.
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What's the use of this, Miss?

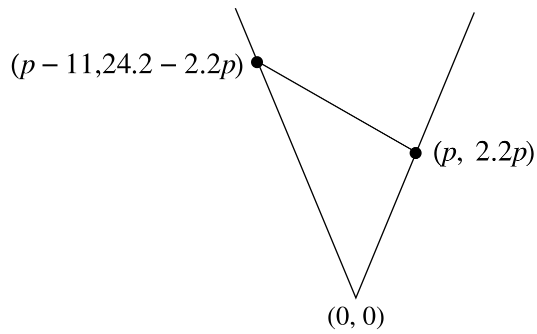
John Searl, Edinburgh Centre for Mathematical Education, University of Edinburgh

At a workshop at a recent conference it was commented that it is often difficult to give an adequate justification to pupils for the mathematics that they are being taught and that this gets more difficult the further up the school one gets. At the S5 and S6 level I would venture to suggest that this is due in part by a reluctance to use simple practical activities. The simple activities below provide motivation, reinforcement of mathematical skills and facts and powerful diagnosers when those facts and skills have not been adequately consolidated. These are well-known results but the pressure of time often squeezes them out of the curriculum. They have all been used in classrooms in recent years (some with primary school pupils!) and have elicited positive reactions from the pupils.

Part 1

1. On an A4 sheet of graph paper, draw a 'Vee' of height 22 cm and width 20 cm. Using the vertex of the Vee as the origin (0, 0) and 1 cm as the unit find the equations of the lines which form the Vee.

2. On your diagram, draw carefully the family of lines which join the point $(p,\ 2.2p)$ to the point $(p - 11,\ 24.2 - 2.2p)$ for $p = 1, 2, 3, \ldots 10$.
   The family of lines are seen to envelope a curve.



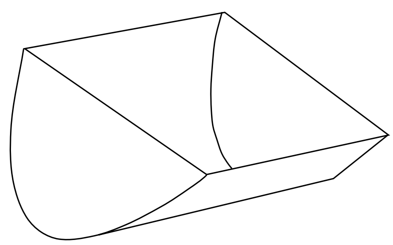
3. Show that the equation of a line in the family of lines is
   $$5y + 11x - 2(x + 11)p + 2p^2 = 0.$$

4. Deduce that two lines of the family pass through every point $(x,\ y)$ on the sheet unless
   $$(x + 11)^2 \leqslant 2(11x + 5y).$$
   What happens if
   $$(x + 11)^2 = 2(11x + 5y)\,?$$

5. Deduce that the equation of the enveloped curve is the parabola
   $$10y = x^2 + 121.$$

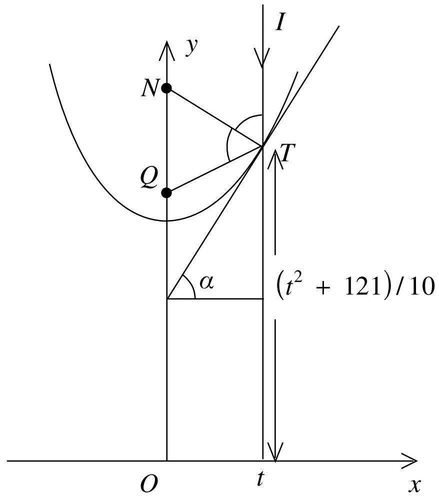
Part 2

1. Use the parabola obtained in part 1 as a template to draw two parabolas on card. Cut them out and construct a parabolic mirror using them and an A4 sheet of card, lining it with acetate.



2. Place the mirror under (and parallel to) a strip light. What do you observe when you put a card along the plane of symmetry of the mirror?

3. Show that the slope of the tangent at the point $\left(t, \dfrac{t^2 + 121}{10}\right)$ on the parabola is $t/5 = \tan\alpha$.



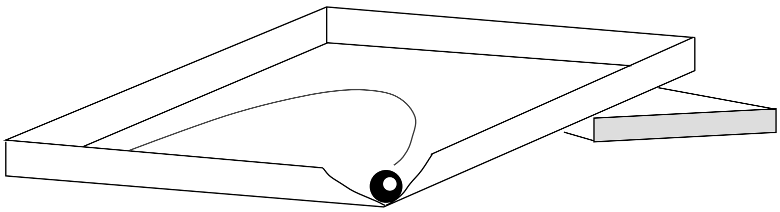
4. A ray of light, parallel to the axis of the parabola, striking the parabolic mirror is reflected through the point Q on the axis as shown in the diagram. The line TN is at right angles to the tangent to the curve at T. The angles $\angle ITN$, $\angle NTQ$ are equal. Show that the line TQ makes an angle 2α with the line QO.

5. Show that $OQ = 14.6$.

6. Explain why all light rays, which are parallel to the y-axis, are reflected through Q.

Part 3

1. Fix some sugar paper inside an A4 box lid. Lean the lid, with its longer side horizontal, on a book or some other support so that the angle of incline is about 20°. Place a marble at a bottom corner of the lid and flick it with your finger. Observe the path of the marble. When you are sure you can obtain a 'good' shape, wet the marble and use it to trace a path. Carefully trace over its trail; with pen or pencil. Describe the curve you obtain.



2. A ball is thrown vertically upwards with initial velocity V m/s. Its velocity $v(t)$ m/s, t seconds later, is given by
$$\frac{dv}{dt} = -g,$$
where $v(0) = V$ and g is the acceleration due to gravity.
Show that $v(t) = V - gt$.

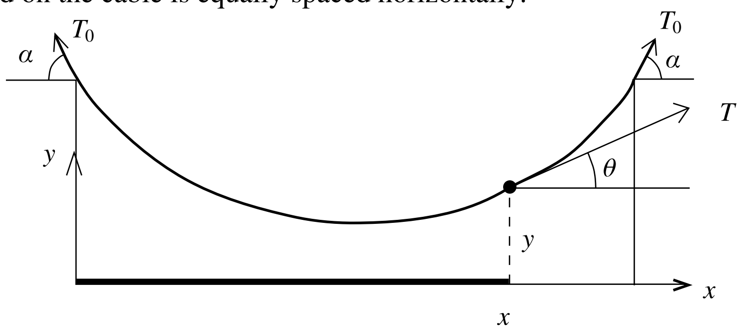
3. Show that the height $y(t)$ m of the ball is given by
$$y(t) = Vt - \tfrac{1}{2}gt^2$$
(This is a parabola in time.)

4. A ball is thrown with initial velocity U m/s horizontally and V m/s vertically. Show that its velocity, t seconds later, is given by
$$\begin{aligned} u &= U && \text{horizontally} \\ v &= V - gt && \text{vertically}. \end{aligned}$$

5. Show that the path traced by the ball is the parabola (in space)
$$y = \frac{V}{U}x - \frac{1}{2}g\frac{x^2}{U^2}.$$

Part 4

1. A suspension bridge is such that the weight of cables is negligible compared to the weight W of the roadway they carry. The actual load on the cable is equally spaced horizontally.
   Show that the tension T_0 in the cable at each end of a symmetrical bridge is given by $T_0 = W/(2 \sin\alpha)$ where α is the angle the cable makes with the horizontal at the end point.



2. Show that the load borne by the cables up to a point distance x from the beginning of a bridge of length l is Wx/l.

3. Show that at the point $P(x, y)$ on the cable the equilibrium of the bridge implies that
$$T \sin\theta = Wx/l - T_0 \sin\alpha$$
$$T \cos\theta = T_0 \cos\alpha$$
   where θ is the angle the cable makes to the horizontal at P.
   Deduce
$$\tan\theta = \frac{2x}{l}\tan\alpha - \tan\alpha.$$

4. Show that the shape of the cable is given by
$$\frac{dy}{dx} = \frac{2x}{l}\tan\alpha - \tan\alpha.$$

5. Verify that $\frac{dy}{dx} = 0$ at the centre of the bridge.
   Find the equation for the line shape of the cable when $y = h$ at $x = 0$ and $y = 0$ at $x = l/2$.

Part 5

1. The standard model adopted for calculating the stopping distance D of a vehicle as a function of its speed V assumes
   (a) there is a fixed reaction time before the driver brakes
   (b) after that, there is a constant deceleration until the vehicle comes to a halt.
   Show that this implies that
$$D = aV + bV^2,$$
   where a and b are constants, and explain the meanings of a and b.

2. The stopping distance is 40 feet at 20 mph and 250 feet at 60 mph. What is the stopping distance when the speed is 40 mph?

3. A driver rounds a bend to see an obstacle 150 feet ahead of him. He is unable to stop before hitting the obstacle. What is the minimum speed at which he could have been travelling?

Comments

These ideas can be developed in many other ways and extended further. For example, the original envelope could be obtained using a graphics calculator. Certainly setting $U = v\cos\theta$ and $V = v\sin\theta$ and using a graphics calculator, enables the pupils to see the effect of varying the angle of launch θ. Doing that they may discover the parabola of safety, another envelope.

I would be pleased to hear from anyone who tries these ideas in the classroom.

What's the use of this, Miss? continued ...

John Searl, Edinburgh Centre for Mathematics Education, University of Edinburgh

Part 1

1. The equations are $y = 2.2x$ and $y = -2.2x$.

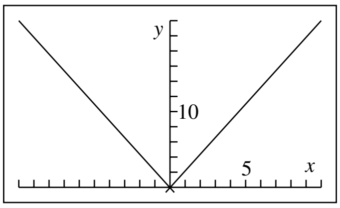


Figure 1:

2.

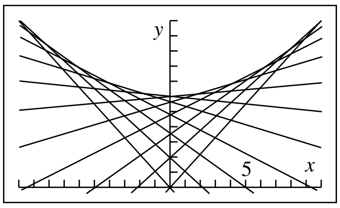


Figure 2:

Drawing by hand is preferable, in my view. The technology is easier to manage!

3. The general line of the family passes though $(p, 2.2p)$ and $(p - 11,\ 24.2 - 2.2p)$. There are many different strategies taught for finding its equation (and long may it continue so!). This time I chose

$$\frac{y - 2.2p}{(24.2 - 2.2p) - 2.2p} = \frac{x - p}{(p - 11) - p}$$

which tidies up to give

$$\frac{y - 2.2p}{24.2 - 4.4p} = \frac{x - p}{-11}$$

and eventually

$$5y = 22p - 2p^2 - 11x - 2px.$$

4. Looking at the drawing, you can see that every point below the envelope has two lines passing through it and above the envelope every point has no lines passing through it. To find the lines which pass through a specific point (X, Y), we have to find the appropriate value of p. That is we have to solve the equation

$$5Y = 22p - 2p^2 - 11X - 2pX.$$

Writing this more conventionally, we have

$$2p^2 - 2(X + 11)p + 5Y + 11X = 0.$$

The condition for this equation to have two different real roots is

$$4(X + 11)^2 > 4(2)(5Y + 11X).$$

So two members of the family pass through (X, Y) unless

$$(X + 11)^2 \leqslant 2(5Y + 11X).$$

When

$$(X + 11)^2 = 2(5Y + 11X)$$

we have equal roots and the two lines coincide and are tangential to the envelope. So the points (x, y) which satisfy

$$(x + 11)^2 = 2(5y + 11x)$$

lie on the envelope, and so we obtain its equation.

5. Simplifying the last equation gives
$$10y = x^2 + 121$$

Part 2

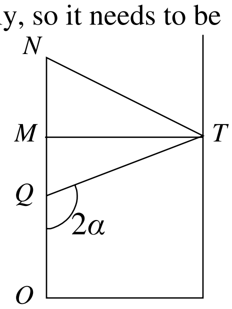
1. This is an activity that students at every level can do and appear to enjoy! The more carefully it is carried out the more satisfying the results obtained in the next step will be. This has sometimes proved a diagnoser of pupils having difficulties of visualisation, of mentally turning a two dimensional plan into a three dimensional object.

2. If the mirror is reasonably well made, a bright line of light will be observed on the card. Distorting the shape of the mirror slightly causes the line to 'vanish'.

3. The equation of the parabola is
$$10y = x^2 + 121$$
so that the slope of the tangent at a general point is
$$\frac{dy}{dx} = \tfrac{1}{5}x$$
where $x = t$ this gives $\tan\alpha = t/5$.

4. The angle $\angle OQT = \angle QTI$ and $\angle QTI = 2\angle NTI$. The angle $\angle NTI = 90° - \angle STI$ while $\angle STI = \angle RTU$. The angle $\angle RTU = 90° - \alpha$ so that $\angle NTI = \alpha$ and $\angle OQT = 2\alpha$.

5. This step is difficult. The $\tan(A + B)$ identity is not taught in S5 usually, so it needs to be developed specially for this example.
From the diagram $OQ = OM - MQ$.
$MQ = MT\tan(\angle MTQ) = t\tan(90 - 2\alpha) = -t/\tan 2\alpha$.Usin
g $\tan 2\alpha = 2\tan\alpha/(1 - \tan^2\alpha)$ we obtain
$\tan 2\alpha = 10t/(25 - t^2)$ since $\tan\alpha = t/5$.
Hence $OQ = (t^2 + 121)/10 + (25 - t^2)/10 = 14.6$.



6. The position of Q is does not depend on t, that is, on the position of T.

Part 3

1. There is scope for some discussion about the result obtained. The symmetry of the trace has implications for the times taken to reach the highest point and to return back to the base line. The meanings of the words 'acceleration', 'velocity' and 'displacement' and the way we use them in a mathematical context need to be teased out.

2. Using the definition of acceleration we obtain the differential equation
$$\frac{dv}{dt} = -g.$$
The formula for the velocity can either be obtained by arguing that constant negative acceleration implies the velocity decreases at a constant rate and the graph of v is a straight line or by formal integration.

3. The relationship that the velocity is the rate of change of height implies
$$\frac{dy}{dt} = V - gt.$$
This can be integrated formally with the initial data that assumes the ball starts at zero height. Alternatively the motion can investigated by means of a displacement-time diagram, evaluating the area under the graph to obtain the distance travelled. This can also be investigated experimentally

using the Texas Instrument CBR or CBL equipment. Such experiments produce a lot of other material to be analysed mathematically.

4. Here there is no horizontal acceleration so the ball's horizontal velocity is unchanged. The vertical velocity follows the same law as in (2).

5. Integrating formally, or otherwise, we obtain

$$x = Ut$$

$$y = Vt - \tfrac{1}{2}gt^2$$

so that $t = x/U$ and $y = Vx/U - \tfrac{1}{2}g(x/U)^2$. It is interesting to investigate how this parabola varies with the angle θ of launch for a fixed launch speed c (running out of letters that do not have conventional meanings here). Then

$$y = x\tan\theta - \tfrac{1}{2}gx^2/(c\cos\theta)^2 .$$

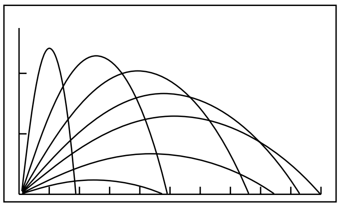


Figure 3:

Using a graphics calculator it is easy to plot the family of parabolas obtained as shown below. Drawing more paths clearly shows that the family is bounded by a parabolic-like curve. Objects outside this curve cannot be hit by the ball without changing the launch speed. Inside the curve we observe that two paths pass through each point, so we have a similar situation to Part 1. Given a point (x, y) how do we determine the launch angle θ to hit it? To find θ we need to solve the equation

$$y = x\tan\theta - \tfrac{1}{2}gx^2/(c\cos\theta)^2 .$$

To find the equation of the enveloping curve we do not need to solve this equation but only to find the condition for it to have equal roots. Because $1 + \tan^2\theta = 1/\cos^2\theta$, this equation is a disguised quadratic equation (in $\tan\theta$). The condition for equal roots is

$$y = \frac{c^2}{2g} - \frac{gx^2}{2c^2}.$$

Part 4

1. There is a difference between the situation where a cable hangs freely under gravity and the situation where a cable supports a road bridge. In the former case the load is the weight of the cable itself and is uniformly spaced along its length. In the latter case, the weight of the cable is very small compared to the weight of the roadway and the load is consequently equally spaced horizontally. It is possible to make a simple model to illustrate this. Using the parabola produced in Part 1 as a template, lay a piece of string or tape along the parabola. Mark points on the string which correspond to 1 cm intervals in the x-direction. Attach paper clips to the string at these marked points and hang up with the ends 20 cm apart. Take another piece of string with the same length and attach the same number of paper clips at equal intervals along its length. Hang that up in the same way and the different line shapes can be easily seen.
Examining the way the forces balance on the bridge, we can see that the weight of the roadway is supported by the (equal) vertical components of the tension T_0 in the cable at the support points. Thus $2T_0 \sin\alpha = W$.

2. The load borne by the cables up to the point distance x from the beginning of the roadway is simply in proportion to that distance.

3. At the point (x, y) on the cable, the balance of forces gives

$$T \sin\theta = Wx/l - T_0 \sin\alpha, \text{ vertically}$$

$$\text{and } T \cos\theta = T_0 \cos\alpha, \text{ horizontally}.$$

Remembering that $W = T_0 \sin\alpha$, by dividing the right-hand and left-hand members of these equations we obtain

$$\tan\theta = \frac{2x}{l} \tan\alpha - \tan\alpha.$$

4. The slope of the cable is $\tan\theta = \frac{dy}{dx}$.

5. Setting $x = l/2$ gives $\frac{dy}{dx} = 0$. Integrating the equation for $\frac{dy}{dx}$ gives

$$y = \frac{x^2}{l} \tan\alpha - x \tan\alpha + C,$$

where C is the constant of integration. The data that $y = h$ at $x = 0$ implies $C = h$. The data that $y = 0$ at $x = l/2$ implies $\tan\alpha = 4h/l$. Hence the lineshape of the cable is

$$y = h\left(\frac{2x}{l} - 1\right)^2.$$

Part 5

1. Let T_1 be the fixed reaction time of the driver, then the vehicle travels a distance VT_1 before the brakes are applied. The time taken thereafter to come to a halt is $T_2 = V/A$, where A is the deceleration, druing which tine it travels a further distance $VT_2 - \frac{1}{2}AT_2^2$. Substituting the value for T_2 gives this further distance to be $V^2/A - \frac{1}{2}A(V/A)^2 = \frac{1}{2}V^2/A$. Hence the total distance travelled before the vehicle stops is $VT_1 + \frac{1}{2}V^2/A$.

2. Data gives

$$20a + 400b = 40$$

$$60a + 3600b = 250$$

from which we obtain $a = \frac{11}{12}$ and $b = \frac{13}{240}$. So for $V = 40$, the stopping distance is $123\frac{1}{3}$ feet.

3. To find the minimum speed we have to solve the quadratic equation

$$aV + bV^2 = 150.$$

With the values of a and b found above, this becomes

$$13V^2 + 220V - 36000 = 0$$

which give the minimum speed to be 44.84 mph. What about the second root … ?

Beginning Arithmetic

Stewart Fowlie   Formerly Principal Teacher of Mathematics, The Edinburgh Academy

A cry has gone up in recent years for a return to basics in the teaching of arithmetic – not mathematics for 5 and 6 year-olds, but arithmetic. For the man in the street, and perhaps for many primary school teachers, this means knowing one's tables and being able to carry out the basic operations of addition, subtraction, multiplication and division on larger and larger numbers without using a calculator, and setting out one's working ‘properly’: we should add being able to do ‘mental’, which means grasping questions read out by the teacher and writing down the answer without showing any working (indeed writing down anything except the answer in deemed to be cheating!).

Once upon a time the ability to add up long columns of figures was daily put to use by most office workers. Shop assistants had to be able to tell their customers what they had to pay, and it was in the interest of the customers to be able to check whether they were being overcharged. Mathematics was only studied by those who stayed on beyond the normal school leaving age.

All is different today – almost all will study some form of mathematics at school, and most will tackle topics which, pre 1939, only arose at university. Virtually all will study some form of science. The clerk is now the technician expected to be able to apply formulae in his work. The shop assistant works at a computer terminal. The customer only has to check that the correct items appear on his bill – he can be certain that the total is correct.

The three Rs of yesteryear have now became Literacy and Numeracy. Literacy still means Reading and Writing, but its purpose is really to do with Communication. Numeracy is still Arithmetic, but what was once seen as ability to calculate is now understanding of number and to use a calculator or computer to do any calculation which cannot be done ‘in one's head’.

Most children will have taught themselves to count before they start ‘proper’ school. A few children are fascinated by number and will teach themselves how to compute mentally with hardly any instruction. It is important to realise that they do this not by memorising ‘tables’ but by developing a structure in their brains which inter-relates numbers.

If all children are to become numerate, they must build such a structure in their minds. Put another way, everything they do in arithmetic must be done with understanding and insight. Anything done because ‘my teacher said ...’ is unlikely to contribute towards the building of the structure. Consider this analogy. Suppose someone comes up to you in the street and asks you the way. If you are familiar with your neighbourhood, you will plan a route in your mind and then say for example, go along that road, take the first on the right, then the second on the left, and so on. Your listener must obey your instructions precisely or he will not find his destination. But your policy to get somewhere is quite different – the way you go may depend on whether you are walking, cycling or driving. But however you go you will be confident of getting to your destination. Solving an arithmetic problem may be done by following the teacher's instructions, or it may be done by ‘knowing the neighbourhood’ – by being numerate. The job of the teacher should be to help the pupil know the neighbourhood. This is not to say that he may be able to point out a short cut, but he should not feel he is doing a good job if he prescribes a route, and makes the pupil go over that route several times so he will never deviate from it in future.

Every task should be set in such a way that every child works out for himself everything. Most number facts will be met sufficiently often that the answer will arise almost instantaneously, but it will be in that part of the brain where there is a framework of number, not stored with arbitrary data like names of things and people. Only in this way will numeracy develop as one works with number.

The notation should be be such that the relation between the numbers written is related to the way they are written. Thus 3 + 5 = ? would not be written as $\begin{array}{r}3\\+5\\\hline\end{array}$ but as $\begin{array}{c}5\\3\ <\ \ ?\end{array}$, 8 – 5 =? as $\begin{array}{c}5\\?\ <\ 8\end{array}$ or as $\begin{array}{c}?\\5 < 8\end{array}$ depending on the question – the former might be ‘Mum gave me 5 coins! now I've got 8 – how many did I have before?’ the latter ‘There were 5 people in the room – now there are 8 – how many have come in?’

One should always start with practical examples, and provide a number line marked like a ruler or perhaps a row of 10 labelled circles (counters!) at the top of the page. Only after the concept is firmly embedded would the question be presented in the numerical form, and even then the pupil should be encouraged to turn each question into a story including the answer. At no time should it be suggested that the facts be memorised – always that they are being worked out. I visualize that most of the work would be 'mental', at least until reading and writing were proficient, though I would not restrict what the pupil wished to write down.

Up to this point it may be noticed that no mention has been made of adding or subtracting, nor of + or – signs. All the pupil has to realise is that he has to count on 5 from 3 to find the answer.

I suggest $3 \overset{5}{<} 8$ may be read in two ways – as '3 and 5 is 8' and as '3 is 5 less than 8'. We can then introduce situations where the first number is greater than the third – for example 'I had 8 coins; now I've only get 3. What has happened?' – I must have spent 5, or given them away. This could be written $8 \overset{-5}{>} 3$ to be read '8 and 5 off is 3' or as '8 is 5 more than 3'. To make sure we remember to put in the ' – ' we might put in ' + ' in the other case, and start calling the signs plus and minus. When all this is second nature we could introduce the notation $3 + 5 = 8$ and $8 - 5 = 3$. Notice that this is not really addition and subtraction but adding positive and negative numbers. We might even find a situation where we can attach meaning to $5 - 8 = -3$, as well as situations where it is meaningless. All this would lead very naturally on to algebra in years to come.

At this stage a calculator might be allowed for checking answers. It is important that the pupil distinguish between $3 \overset{5}{<} ?$ and $5 \overset{3}{<} ?$, the thought processes being different even though the answers are the same. The realisation that the answers are the same must be discovered by the pupil – when he thinks of it, his insight will make it seem obvious that numbers behave like that.

Decimal notation can be explained by thinking about people counting with their fingers. 26 means 2 people's fingers and 6. (The commutative rule is implied here – it's the same as 6 + 2 people's!) Also the marking of cm and mm on a ruler to represent 2.6cm as being an alternative way to writing 26 mm introduces the decimal point. In due course a second decimal place comes from considering £ and pence. It will be noticed that this comes before there is any reference to (vulgar) fractions.

It will be realised that adding a small number to a large number is easier than the reverse! What we call the associative rule for addition must also be discovered by the child. This concept (with the commutative rule) is needed before we can justify adding three quantities together. Notice that we should expect for example 8 + 6 to be found by using 2 out of 6 to make 10, and the remaining 4 of the 6 gives the final answer as 14. There must also be discussion of adding two different things together 2 cm + 6 mm doesn't make 8 of anything. But 2 sheep and 6 cows do make 8 animals. 1 cup of milk + 1 cup of milk may make 1 jug of milk – lots to talk about.

Doubling should be introduced before general multiplication. The notation 2(5) seems more natural than 2×5, and will again lead more naturally to algebra in due course, and 2(5) is the same as 5 + 5 and is called double or twice 5. Again practical situations should be involved – You've got 5 fingers on one hand – how many have you got on both hands? My friend and I shared out some sweets between us – if we each got 4 sweets, how many were there altogether?

Then 4(5) is just double double 5. 3(5) is double 5 and 5, 5(5) is double double 5 and 5, 6(5) is double double 5 and double 5 (or perhaps double 3(5) or double (double 5 + 5)). 7(5) is double double 5 + double 5 + 5. 8(5) is just double double double 5, 9(5) is just double double double 5 + 5, or perhaps 10(5) – 5 once it is realised that 10(5) is obviously 50. The important thing is for the child to feel that there are many ways of finding a particular answer, not just one 'right' way as prescribed by the teacher. A useful practical situation would be to consider the number of (square) tiles in a given rectangular array, which will lead the pupil to realise that 4(5) has the same answer as 5(4).

Division may be related to this as ?(4) = 12 (4 + 4 = 8, + 4 = 12). A suitable question might be: there are 12 children in a class – they sit at tables in fours – how many tables are needed? It is probably better not to formalise ‘remainders’ to begin with, but to think of the meaning in each case. In the case of 10 children sitting at tables, we would need 4 tables, but there would only be 2 sitting at the last one, or perhaps 3 at each of the last two. Thus ?(4) = 14 would have the answer 4. But if the question were about how many teams of four a class of 14 could form, the answer would be 3, with a reserve for two of the teams.

Notice that the question 3(?) = 12 will seem to have a different meaning – If 12 children are to be divided into 3 teams, how many would be in each? In English, we ‘divide into’, and that does not mean the same as ‘divide by’ in arithmetic. It may be better to avoid the mathematical use of the word at least at first. However the use of ‘half’, ‘third’ might be introduced and meaning attached to 1/2(12) = 6.

Again answers once found may be checked with a calculator, any figures after the decimal being quietly ignored.

Formal methods of setting out addition, subtraction, multiplication or division at once run the danger of turning the procedure into a meaningless ritual. It may be that they need never be introduced, because in the future a calculator will be used in all cases when the formal algorithm is required. The use of a calculator in due course will be to evaluate values given by formulae, and problems should be solved by calculator written first in that form e.g. Find 1/3(36) – 1/4(20).

Many teachers may criticize the above approach because questions will take longer to do. However every moment spent will be spent thinking in a mathematical way, as opposed to merely carrying out procedures. I had a pupil who produced an obviously wrong answer to a question. On this being pointed out, he said ‘but I did what I was told’. With the correct approach, a pupil may write down irrelevant working, but he should never write down nonsense.

Others may say it is obvious that I have little experience of teaching young children. This is true but I have plenty of experience of working with older children and finding lack of understanding of basic ideas – often in the relatively able who found it easier to memorise than to learn with understanding – the kind who ask for help saying ‘I can't remember how to do this sort of question’ rather than ‘I don't understand this question’. It is the philosophy behind the approach which is important, not the detailed syllabus.

Book Reviews
Edited by Mary Teresa Fyfe
Principal Teacher of Mathematics, Hutchesons' Grammar School, Glasgow

Maths Now 3 and 4 (Green orbit)
ISBN 071957352 1 Student Book £8.99
ISBN 071957354 8 Student Book £8.99
ISBN 071957353 X Teacher's Resource File £35
ISBN 071957355 6 Teacher's Resource File £37.50 (John Murray)

These books form part of in a series aimed at students with special educational needs in Secondary Schools. There is a starting placement test to check which chapters are necessary for individual pupils, to be followed by many examples on each topic and some excellent worksheets for reinforcement. At the end of each chapter there is a test on that particular section of work as well as a general review. The vocabulary used in each section is stressed and homework pages are included. This book would be an excellent source of material for those pupils with special educational needs.

Maths Now 2 (Red orbit)
ISBN 071957097 2 Student Book £10.99
ISBN 071959098 0 Teacher's Resource File £37.50 (John Murray)

This book is part of a series aimed at the average and above average pupil. Each chapter begins with an interesting and helpful introduction and finishes with useful supplementary exercises, but, by limiting the number of questions in each exercise to at most ten, extra resources would be necessary. The inclusion of a mental arithmetic chapter at the end of the teacher's file is very welcome. From the Scottish viewpoint some topics are covered which are not in Standard grade, such as vectors. This book would be a useful resource of introductory work and perhaps consolidation but not as a main textbook. However, I have found the reproducible material of much value in my own classroom.

P. Chisholm
Castlehead High School

SMP 11-16 Book A9
ISBN 0 521 47848 0 £4.95
Cambridge University Press

Perhaps this review should be titled *Confessions of an Amber User*. Having bought Amber 1- 8, I am very much attached to the series, whilst still not sure whether I love it or hate it. We use it in S3 and S4 with classes for whom Foundation would be a challenge, and it certainly caters for them.

There is a lot I like about Amber. It is well illustrated with colourful photos and pictures. It has a lot of good work in relevant and interesting situations. There are some good games and drawing activities, and the Review questions at the end of each book are useful for checking pupil progress, or even a quiet five minutes.

There is a lot I hate about Amber. £4.75 is not a lot for a book, but A9 has 49 pages! Lots of the sections in Amber are excellent but most of them are less than five pages and you don't get many questions on a page, never mind an exercise. In A9 there is a section on Paying Taxes. The part on VAT continues on a worksheet (lots of worksheets in Amber!). In the book there are 3 questions and one of them is

"Which sorts of things do you think should be free of VAT?"

I do not mind supplementing the series to give a full depth of coverage for Foundation, but would prefer if there were more useable questions. Some of the sections make me shudder and I pass them by, but then I am so old I still use chalk, and lots of it.

A9 is the last in the Amber series. If you like and use A1 to 8 you will want A9. If you have pupils for whom grades 6 or 5 will be a struggle then you should have a good look at Amber.

T. Leach
Newbattle High School

Maths in Action Intermediate Mathematics Series Nelson
Intermediate 1 Mathematics
Pupil Book ISBN 0 17 431497 3 Teacher's Resource Book ISBN 0 17 431495 7
Intermediate 2 Mathematics
Pupil Book ISBN 0 17 431494 9 Teacher's Resource Book ISBN 0 17 431498 1

At last, teacher's prayers have been answered. These pupil books and resource packs fill a major gap in the provision of good material in book form for Fifth Year students. The style and layout of each pupil book is perfect for the older secondary school student. The syllabus content is thoroughly covered. The exercises are of appropriate length, providing enough consolidation of topics if required. Clear guidance is given by printing harder questions against a coloured background. The chapters align well with the syllabi and the Revision chapter at the end of each unit will be very helpful for teacher and pupil alike.

The teacher's resource packs supply many extra examples in a similar style to the pupil book and provide the hard-pressed teacher with more than adequate back-up material.

It would be churlish to ask for more when so much has been done by the authors to provide an excellent textbook for what is a large part of the pupil population for post-Standard Grade Mathematics, but it would have been nice to have some practice for Unit tests. Overall, these books will provide Scottish students for many years to come with a fine resource to help obtain a good examination result.

R Mc Kenna
Saint Columba's High School

Introducing Statistics
Graham Upton and Ian Cook
ISBN 0 19 914561 X £12.50
Oxford University Press
This book is part of the well-known *Introducing and Understanding* series from these publishers. It is as good a textbook as the others in the series for A Level students. It covers all topics in the A level modules for Statistics as part of a Mathematics A Level. Although it is lacking in a couple of topics from the Advanced Higher, it could still form the basis for teaching that course.
Each chapter covers a single topic, subdivided into short learning outcomes. There are plenty of examples for each outcome – perhaps more than most students require. The introduction to each exercise provides the student with much background knowledge to supplement the classroom teacher's notes. The chapter summary gives a concise outline of what has been covered in the chapter and there is a plentiful supply of additional examples covering a whole topic.
This book is the best I have seen for A level students, providing an excellent support for the classroom teacher.

Understanding Statistics
Graham Upton and Ian Cook
ISBN 0 19 914391 9 £12.50
Oxford University Press
This book supplements the shorter *Introducing Statistics* to meet the needs of students of A level Statistics and first year undergraduates. It includes all of the material from the simpler book and covers more advanced material to provide a lively context and interesting examples of the uses of Statistics.
The format is identical to that of *Introducing Statistics*.
Given that topics such as Shewart control charts and the Mann-Witney test are included here, this book would provide a teacher of Advanced Higher Mathematics with excellent background material to use with their class.
Each of these two books are reasonably priced and supply the needs of the teacher of Statistics in sixth year.

M T Fyfe
Hutchesons' Grammar School

Oxford Mathematics for GCSE
Oxford University Press Foundation GCSE
ISBN 0 19 914717 5 £13
Higher GCSE ISBN 0 19 914707 8
These books form part of a three book series (there is one for Intermediate as well) which cover the two year GCSE Course. However, the series should not be discarded by Scottish teachers because there is much here which can be of use in the Standard Grade classroom.
The books are attractively presented, in full colour, and with many illustrations which cover teaching points as well as interesting 'real life' situations. The texts are very well structured, offering good guidance for pupils about what has to be learned in that worked examples and learning points are always to be found in a yellow box. There are more than enough examples to ensure competency for the learner and there are regular *Skills Breaks* and *Review* sections which test understanding of work covered in a section. Another useful feature is the start of each chapter being a listing of what the pupil should already know and the end by a list of what should have been learned. Perhaps the most innovative and useful feature of this series is an index, *Wordfinder* section which directs the user to examples and information about every mathematical context covered in the book. This index would prove invaluable to both teacher and pupil alike.
Although the Foundation book contains more material than is required for the Standard Grade Foundation syllabus, there is certainly much that a General pupil can get their teeth into. Although supplementary work on similarity, the straight line, trigonometry, the circle and variation would be required, the majority of these topics are well covered in the Higher GCSE text. This book would be extremely useful for Credit pupils because it approaches this syllabus in a stimulating and comprehensive way. This book would only require minor supplementation for calculations involving arcs and sectors and trigonometrical relationships.
These books are well worth a look because they provide a novel yet useful coverage of much of the Scottish syllabi. Their attractive presentation and plethora of examples as well as their organised structure would provide a welcome relief from the commonly used texts.

K Russell
Mainholm Academy

Caculator Maths from A+B Books

There are five books in the series which link the use of graphing calculators into four specific areas of the curriculum. These are Number, Algebra, Shape and Handling Data. The fifth book, called Foundations, deals with the basic operation of the machine and some specific calculator skills required for work in the other books. The books are designed to be used with TI-80 machines and make use of one or two of the specific functions available on this machine. We must confess that we are not big fans of the TI-80 and we were originally put off these books by the choice of machine. This was a mistake; there are some real gems here which we both have used in the classroom. Most (if not all) the ideas can be used with other models of TI machines and could easily be adapted for use with other manufacturers' machines.

Many of the ideas were a new departure for most graphing calculator books in that there is a genuine attempt to tackle many of the fundamental ideas we teach. We were particularly impressed with the way Number is tackled. The current push towards numeracy would caution against using a calculator but this book suggests otherwise. Topics such as equivalent fractions, calculating with fractions, decimals and integers are all tackled with an imaginative approach, the calculator being used as a tool for discovery rather than as a calculating machine. This has led to fun in an often dry numeracy lesson. The book also looks at standard form, and hence can be used as a resource across the school, not just in specific year groups. To our mind this makes them a valuable resource. It is not often you buy textbooks which can span the year groups.

We were intrigued to find Algebra as a topic for one of the books, wondering how it would be tackled on a non-Computer Algebra System (CAS) machine. Here was one of the first gems (and something we have been kicking ourselves ever since for not thinking of!). The basic idea is that you store a number to a variable name then enter an equation using this variable, successively 'stripping' away the additive constants and coefficients while evaluating at each step. This essentially takes you through the process of solving the equation; a new approach which my colleague and I have both tried in our classrooms with great success.

Shape is perhaps our least favourite since we do not do much of what is in the book. The 'similar shapes' section worked well as a classroom lesson driven by the teacher on the overhead version of the calculator. As pupils 'work through' it is uncertain how it would have gone as it requires some quite advanced use of the machine.

By far the most useful book in the series is Handling Data and if you have to buy only one, buy this one. This book covers a great deal of the statistics introduced at Standard Grade and into the 5-14 documents as well as some material for Intermediate Two. Its greatest strength is perhaps that it introduces the user to statistics on the machine and allows you to explore and interpret many data sets without the tedium of long calculation. With the focus on interpretation of statistics in Higher Still, this is a must.

Many people will recognise the authors (Alan Graham and Barrie Galpin) from the 'Tapping into' series of books for the Open University. Over a number of years they have obviously invested a great deal of time in getting to know the strengths of the technology and where best to apply it. These books are the obvious fruits of their labours. I have often heard, 'I would use the graphing calculator in my class but there are no resources'. Well these books solve that; there is something in each book which is useful right away with little preparation. With so few useful resources out there these are a must for anyone looking to start using graphing calculator technology in the classroom.

Derek Simpson, Beeslack CHS and
Calum Stewart, Preston Lodge High School.

Maths Workout for homework and Practice Books 1 2 and 3

Bob Hartman and Mark Patmore
Book 1 ISBN 0 521 62489 X £3.50
Book 2 ISBN 0 521 63488 1 £3.50
Book 3 ISBN 0 521 63487 3 £3.50
Cambridge University Press

These small, well-presented books offer a colourful variety of exercises and tasks for younger pupils (aged 11 - 14) to revise and consolidate their skills. Each section starts with a set of *key ideas* to indicate what previous knowledge is required. The section is then divided into three sets of questions of varied degree of difficulty. There is much emphasis of mental arithmetic and there are also many questions which involve the user in 'real life' contexts. However, the lack of guidance given in the 'investigation' exercises would require much supplementation from the teacher, e.g. 'Look around your house and near where you live. Try to find where the old Imperial units are used.'

Although clear indication of the topic for practice is given at the start of each section, the sheer variety of question and the emphasis on the pupil's own gathering of informafion would entail the teacher in much preparation.

J. O'Toole
Woodfarm High School

Achievement in Mathematics - Revision and Practice

M. E. Wardle and A. Ledsham

ISBN 0 19 914736 1 £10
Oxford University Press

The style of this book will be familiar to those who have seen the revision and practice books for GCSE. Here the content is aimed at the most basic level of the National Curficulum. The layout is clear and concise, traditional in format and therefore relatively unexciting. The level of question is appropriate to the bottom end of the ability range and, although there are some illustrations, there are many pages of straightforward examples with little distraction for a pupil with limited concentration. This book should find a ready market for those teachers who think repetition leads to success. As a textbook for students aiming at a basic numeracy qualification, this resource has much to offer.

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