5: Dividing Whole Numbers by Fractions

**Question:** What is the value of \( \frac{3}{4} \)?

<table>
<thead>
<tr>
<th>Misconception</th>
<th>Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>The value of ( \frac{3}{4} ) is equivalent to ( \frac{3}{4} ) and hence has value ( \frac{3}{4} ) or 0.75.</td>
<td>The division ( \frac{3}{4} ) means how many ( \frac{1}{4} ) are there in the number 3. Clearly there are 4 quarters in 1 and hence ( 3 \times 4 = 12 ) in 3. So ( 3 \div \frac{1}{4} = 12 )</td>
</tr>
<tr>
<td>Similarly ( 5 \div \frac{1}{2} ) is equivalent to ( 5 \div 2 ) and hence has value 2.5 or ( 2 \frac{1}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

**Further Explanation**

We must learn NEVER to be influenced by what things look like: the meaning of dividing by 2, dividing by 5, etc. is clear: the concept of dividing by a quarter is, however, less straightforward and requires more thought.

Think of \( 3 \div \frac{1}{4} \) as 'the number of \( \frac{1}{4} \)'s that fit into 3'.

There are 4 quarters in 1, so in 3 there are \( 3 \times 4 \) quarters in 3 as can be seen in the diagram below.

\[
\begin{array}{ccc}
1 & 1 & 1 \\
\frac{1}{4} & \frac{1}{4} & \\
\frac{1}{4} & \frac{1}{4} & \\
\end{array}
\]

So \( 3 \div \frac{1}{4} \) (or \( \frac{3}{\frac{1}{4}} \)) = \( 3 \times 4 = 12 \).

Generally \( n \div \frac{1}{m} = n \times m \) or \( \frac{n}{\frac{1}{m}} = n \times m \)

Hence, for example, \( 5 \div \frac{1}{2} = 5 \times 2 = 10 \).
Misconception 5

There is another way to approach this task logically which we will demonstrate with $6 \div \frac{3}{5}$.

Use the problem solving method – 'if you are having difficulties, find something similar which you know you CAN do and work out the difference between this and the problem given'.

The difficult part here is dividing by a fraction.

Start with something similar which is straightforward: just divide the 6 by $3 \left(\frac{6}{3}\right)$. Now continue by examining the effect of the difference between what we did and what was given (using clearer terminology to refer to division, i.e. divide between).

We divided the 6 by 3 instead of by the given $\frac{3}{5}$ (which is, of course, less than 3).

When a cake is divided between a certain number of people, each gets a certain portion. Dividing it between fewer people results in each one receiving a larger portion. How much larger? If it is divided between, say, 5 times fewer people, each portions would become 5 times larger.

We arrived at 2 by dividing the 6 by 3. We should have divided by something that is 5 times smaller than the 3, (by $\frac{3}{5}$), so, the result should be 5 times larger than the $\frac{6}{3}$. Thus we deduce that our $6 \div \frac{3}{5}$ must mean $\frac{6}{3} \times 5$ (= 10). Generalising,

$$a \div \frac{b}{c} \text{ (or } \frac{a}{\frac{b}{c}}\text{)} = \frac{a}{b} \times c \left(= \frac{a \times c}{b} = \frac{a \times c}{b}\right)$$

Yet another way of determining $\frac{3}{4}$ is to forget about the unclear meaning of dividing by a fraction and to do whatever yields a result which doesn't contradict other things that are already established.

Whatever we mean by $\frac{p}{k}$, we already know that its result, $r$, must be such that $r \times k$ will be equal to $p$. i.e. in $\frac{p}{k} = r$, $r$ must be such that $r \times k = p$, (e.g. $\frac{187}{11}$ is 17 because $17 \times 11 = 187$).

Following this for $\frac{3}{4}$, we simply seek a result which gives 3 when multiplied by a $\frac{1}{4}$.

The question then becomes: "what times a quarter is 3?", or using a familiar rephrasing "a quarter of what is 3?" (The answer is of course 12.) In summary

$$\text{to determine the value of } r \text{ in } \frac{3}{4} = r, \text{ find which value of } r \text{ satisfies } r \times \frac{1}{4} = 3$$
Follow-up Exercises

1. Calculate the value of:
   
   (a) $4 \div \frac{1}{2}$  
   (b) $3 \div \frac{1}{3}$  
   (c) $6 \div \frac{1}{4}$  
   (d) $10 \div \frac{1}{5}$

   (e) $4 \div \frac{1}{3}$  
   (f) $5 \div \frac{1}{4}$  
   (g) $20 \div \frac{1}{5}$  
   (h) $6 \div \frac{1}{6}$

2. Calculate the value of:

   (a) $4 \div \frac{2}{5}$  
   (b) $3 \div \frac{3}{4}$  
   (c) $10 \div \frac{2}{3}$  
   (d) $4 \div \frac{3}{4}$

   (e) $6 \div \frac{3}{5}$  
   (f) $1 \div \frac{2}{5}$  
   (g) $7 \div \frac{5}{8}$  
   (h) $20 \div \frac{4}{5}$

Answers

1. (a) 8  (b) 9  (c) 24  (d) 50  (e) 12  (f) 20  (g) 100  (h) 36

2. (a) 10  (b) 4  (c) 15  (d) $\frac{16}{3}$  (e) 10  (f) $\frac{5}{2}$  (g) $\frac{56}{5}$  (h) 25