7: Calculations with Negative Numbers

**Question:** What is the value that satisfies \(-4 + ? = -10\)?

<table>
<thead>
<tr>
<th>Misconception</th>
<th>Correct</th>
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<tbody>
<tr>
<td>There are several possible misconceptions including (? = 6)</td>
<td>The correct answer is (-6) as this satisfies the sum (-4 + (-6) = -10). It can be shown on a number line as given below.</td>
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<tr>
<td>and (? = 14)</td>
<td>(-10) (-8) (-6) (-4) (-2) 0 (+2) (+4) (+6)</td>
</tr>
<tr>
<td>or even (? = -14)</td>
<td>(\leq) ((-6))</td>
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<tr>
<td>Starting at negative 4 and adding negative 6 you get the answer negative 10.</td>
<td></td>
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<tr>
<td>Hence (-4 + (-6) = -10)</td>
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**Further Explanation**

As with the previous misconception (number 6), it is important to realise that there is no guessing, no doubts but straightforward logic. Here we can associate the meaning *owning* when the quantity is positive and *owing* if it negative.

Also, as before, we associate the *minus* sign with *opposite*.

In this example,

I start by owing 4 (i.e. the ‘\(-4\)’)

What is this ‘+?’ that must have happened if I ended up ‘owing 10’? (i.e. the ‘\(-10\)’)

Another *debt* of 6 must have been added to my plight

We write 'adding a debt of 6' as ‘\(+(-6)\)’

So in \(-4 + ? = -10\) the ? must be \(-6\)

This is equivalent to the use of a number line as shown above; it does not matter which way you argue as long as you get the logic correct!
Follow-up Exercises

You might find a number line helpful when making or checking your calculations.

1. Complete the following:
   (a) \(-5 - 3 = \square\)
   (b) \(-7 - 2 = \square\)
   (c) \(-9 - 8 = \square\)
   (d) \(-4 - 4 = \square\)
   (e) \(-3 - 8 = \square\)
   (f) \(-5 - 9 = \square\)

2. Complete these calculations:
   (a) \(-5 + \square = -9\)
   (b) \(-3 + \square = -7\)
   (c) \(-7 + \square = -10\)
   (d) \(-8 + \square = -6\)
   (e) \(-4 + \square = -1\)
   (f) \(-10 + \square = -6\)

3. Calculate the value of each of these expressions:
   (i) \(a + b\)  (ii) \(a - b\)  (iii) \(-a + b\)  (iv) \(-a - b\)
   when
   (a) \(a = -3\) and \(b = -7\)
   and
   (b) \(a = 5\) and \(b = -6\)

Answers

1. (a) \(-8\)  (b) \(-9\)  (c) \(-17\)  (d) \(-8\)  (e) \(-11\)  (f) \(-14\)
2. (a) \(-4\)  (b) \(-4\)  (c) \(-3\)  (d) \(2\)  (e) \(3\)  (f) \(4\)
3. (a) \(-10, 4, -4\) and \(10\)  (b) \(-1, 11, -11\) and \(1\)